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Mixed Problem for one Class Semilinear Fourth-order Hyperbolic Systems

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Abstract. Mixed problem for systems of fourth-order semilinear hyperbolic equations is considered, when fourth and second-order derivatives with different variables participate in each equation. The theorem on local solvability is proved. Further, for a class of semilinear fourth-order systems, the theorem on solvability "as a whole" is proved.

Key Words and Phrases: semilinear hyperbolic equation, mixed problem, solvability

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1. Statement of the problem and the main result on local solvability

Consider the systems of semilinear hyperbolic equations of the fourth order

$$\left.\begin{array}{l} u_{1tt}+\Delta_{I_1}^2 u_1-\Delta_{J_1} u_1=f_1\left(t,x,u_1,u_2\right), \\ u_{2tt}+\Delta_{I_2}^2 u_2-\Delta_{J_2} u_2=f_2\left(t,x,u_1,u_2\right), \end{array}\right\} \tag{1}$$

where $\Delta_{Ik}=\sum_{i\in I_k}\frac{\partial^2}{\partial x_i^2}$, $\Delta_{Jk}=\sum_{i\in J_k}\frac{\partial^2}{\partial x_j^2}$, $I_k\subset N_n=\{1,...,n\}$, $J_k=N_n\backslash I_k$, $k=1,2$.

Let us denote by $m_r=\overline{\overline{J_r}}, r=1,2$ the number of elements J_r, and denote by $n_r=\overline{\overline{I_r}}=n-m_r$ the number of elements I_r. For definiteness, suppose that

$$m_1\leq m_2. \tag{2}$$

A system of type (1) is encountered in the study of oscillations of deformed systems under moving loads (see e.g. [1]). In [2], the Cauchy problem for systems of equations

$$\left.\begin{array}{l} u_{1tt}+\Delta_{I_1}^2 u_1-\Delta_{J_1} u_1+u_{1t}=g_1\left(u_2\right), \\ u_{2tt}+\Delta_{I_2}^2 u_2-\Delta_{J_2} u_2+u_{2t}=g_2\left(u_1\right), \end{array}\right\}$$

is investigated, where

$$|g_1(u_2)|\leq|u_2|^p, \quad |g_2(u_1)|\leq|u_1|^q,$$

and found conditions for the growth of coefficients p, q ensuring the existence of global solutions.

In this paper we study the existence and uniqueness of local and global solutions of a mixed problem for the system (1).

Let $\Pi = [0,1]^m$. In the cylinder $Q_T = [0,T] \times \Pi$ we consider the mixed problem for system (1) with the boundary conditions

$$u_k\left(t, x_1, x_2, ..., x_i, ..., x_n\right) = 0, \qquad x_i = 0 \;, x_i = 1, k = 1,2\,; \quad i = 0,1,...,n\,; \tag{3}$$

$$\Delta_{I_k} u_k\left(t, x_1, x_2, ..., x_i, ..., x_n\right) = 0, \qquad x_i = 0 \;, x_i = 1, i \in I_k, \quad k = 1,2\,; \tag{4}$$

and the initial conditions

$$u_k\left(0, x\right) = \varphi_k\left(x\right), \;\; u_{kt}\left(0, x\right) = \psi_k\left(x\right), \;\; x \in \Pi, \;\; k = 1,2. \tag{5}$$

By $W_{2,k}^{2s,s}$, $k = 1,2$ denote the functional space with a finite norm:

$$\|u\|^2_{W_{2,k}^{2s,s}} = \left\{ \|u\|^2_{L_2(R_N)} + \sum_{i \in I_k} \left\|D^{2s}_{x_i} u\right\|^2 + \sum_{j \in J_k} \left\|D^{s}_{x_j} u\right\|^2 \right\}^{\frac{1}{2}},$$

and by H denote the functional space $\mathrm{H} = W_{2,1}^{2,1} \times L_2(R^n) \times W_{2,2}^{2,1} \times L_2(R^n)$, with the scalar product

$$\left\langle w^1, \; w^2 \right\rangle_{\mathrm{H}} = \int_\Pi \Delta_{I_1} v_1^1 \;\; \Delta_{I_1} v_1^2 dx + \int_\Pi \nabla_{J_1} v_2^1 \, \nabla_{J_1} v_2^2 dx +$$

$$+ \int_\Pi \Delta_{I_2} v_3^1 \;\; \Delta_{I_2} v_3^2 dx + \int_\Pi \nabla_{J_2} v_4^1 \, \nabla_{J_2} v_4^2 dx.$$

Let us assume that

$$n + m_2 < 4. \tag{6}$$

and the following condition holds.

I. The functions $f_1\left(t, \, x, \xi, \, \eta\right)$ and $f_2\left(t, \, x, \xi, \, \eta\right)$ are defined for all $t \in [0,T]$, $x \in \Pi$, $\xi \in R$, $\eta \in R$ and continuously differentiate with respect to t, ξ and η;

The following theorem on local solvability is proved.

Theorem 1. *Let* (2), (6) *and the condition* ***I*** *be satisfied. Then for any* $(\varphi_1, \psi_1, \varphi_2, \psi_2) \in$ H *there exists* $T' > 0$ *such that the problem* (1), (3) - (5) *has a unique solution* $(u_1, u_2) \in C\left([0,T']\,; W_{2,1}^{2,1} \times W_{2,2}^{2,1}\right) \bigcap C^1\left([0,T']\,; L_2(\Pi) \times L_2(\Pi)\right)$.

Moreover, if $T_{\max}$ *- the length of the maximal interval of the existence of a weak solution is* $(u_1, u_2) \in C\left([0, \, T_{\max})\,; W_{2,1}^{2,1} \times W_{2,2}^{2,1}\right) \bigcap C^1\left([0, \, T_{\max})\,; L_2(\Pi) \times L_2(\Pi)\right)$, *then one of the following statements hold:*

i) or $T_{\max} = +\infty$,

ii) or $\lim\limits_{t\to T_{\max}-0} E(t) = +\infty$, *where*

$$E(t) = \sum_{k=1}^{2} \left[\; \|u_k(t,\cdot)\|_{W_{2,k}^{2,1}} + \left\|u'_{kt}(t,\cdot)\right\|_{L_2(R^n)} \right], \quad t \in [0, T_{\max}).$$

If additionally $(\varphi_1, \psi_1, \varphi_2, \psi_2) \in W_{2,1}^{4,2} \times W_{2,1}^{2,1} \times W_{2,2}^{4,2} \times W_{2,2}^{2,1}$, *then*

$$(u_1, u_2) \in C\left([0,T']; W_{2,1}^{4,2} \times W_{2,2}^{4,2}\right) \bigcap C^1\left([0,T']; W_{2,1}^{2,1} \times W_{2,2}^{2,1}\right) \bigcap$$

$$\bigcap C^2\left([0,T']; L_2(R^n) \times L_2(R^n)\right).$$

Now consider the case $n + m_2 \geq 4$. In this case, the following condition on the growth of nonlinearity is required. First let us consider the case $n + m_1 < 4$

II. a) Let $n+m_1 < 4$, $n+m_2 \geq 4$ and for all $(t,\, x,\, \xi,\, \eta) \in [0,\mathrm{T}] \times \Pi \times R^2$ the following estimates are fulfilled

$$|f_k(t,x,\xi,\eta)| + \left|f'_{kt}(t,x,\xi,\eta)\right| \leq c(\xi)\left[g_k(x) + |\eta|^{p_k}\right],$$

$$\left|f'_{k\xi}(t,x,\xi,\eta)\right| \leq c(\xi)\left[h_k(x) + |\eta|^{q_k}\right],$$

$$\left|f'_{k\eta}(t,x,\xi,\eta)\right| \leq c(\xi)\left[h_k(x) + |\eta|^{q_k}\right],$$

where $c(\cdot) \in C(R;\; R_+)$, $g_k(\cdot) \in L_2(\Pi)$,

$$\text{if } n + m_2 = 4 \quad \text{then } \ p_k \in [1, \infty) \quad \text{and if } \ n + m_2 > 4 \quad \text{then } \ p_k \leq \frac{n+m_2}{n+m_2-4}.$$

II b) Let $n + m_1 \geq 4$ and for all $(t,\, x,\, \xi,\, \eta) \in [0,\mathrm{T}] \times \Pi \times R^2$ satisfy the following estimations

$$|f_k(t,x,\xi,\eta)| + \left|f'_{kt}(t,x,\xi,\eta)\right| \leq c\left[g_k(x) + |\xi|^{p_{k1}} + |\eta|^{p_{k2}}\right]$$

$$\left|f'_{k\xi}(t,x,\xi,\eta)\right| \leq c\left[h_k(x) + |\xi|^{q_{k1}} + |\eta|^{q_{k2}}\right];$$

$$\left|f'_{k\eta}(t,x,\xi,\eta)\right| \leq c\left[l_k(x) + |\xi|^{r_{k1}} + |\eta|^{r_{k2}}\right],$$

where $c \in R_+ = [0, \infty)$, $g_k(\cdot) \in L_2(\Pi)$, and $h_k(\cdot)$,$l_k(\cdot)$, p_{ki} and q_{ki} satisfy the following conditions:

$b_1)$ $h_k(\cdot), l_k(\cdot) \in L_p(\Pi)$, $p \in (2,\infty)$, $p_{ki}, q_{ki} \in (1,\infty)$, $i = 1,2;$ $k = 1,2$, if $n + m_1 = n + m_2 = 4$;

$b_2)$ $h_k(\cdot) \in L_p(\Pi)$, $p \in (1,\infty)$, $l_k(\cdot) \in L_{\frac{n+m_2}{2}}(\Pi)$, $k = 1,2$,

$$p_{k1}, \quad q_{k1}, \; r_{k1} \in (1,\infty), p_{k2} \leq \frac{n+m_2}{n+m_2-4}, \tag{7}$$

$$q_{k2} < \frac{n+m_2}{n+m_2-4}, r_{k2} \leq \frac{4}{n+m_2-4}, \quad k = 1,2, \tag{8}$$

if $n+m_1=4, \quad n+m_2>4$;
$b_3)$ $h_k(\cdot)\in L_{\frac{n+m_1}{2}}(\Pi), l_k(\cdot)\in L_{\frac{n+m_2}{2}}(\Pi)$, $k=1,2,$

$$p_{k1}\le \frac{n+m_1}{n+m_1-4},\ p_{k2}\le \frac{n+m_2}{n+m_2-4},\quad k=1,2, \tag{9}$$

$$q_{k1}\le \frac{4}{n+m_1-4},\ q_{k2}\le \frac{4(n+m_2)}{(n+m_2-4)(n+m_1)},\quad k=1,2, \tag{10}$$

$$r_{k1}\le \frac{4(n+m_1)}{(n+m_1-4)(n+m_2)}, r_{k2}\le \frac{4}{n+m_2-4},\quad k=1,2, \tag{11}$$

if $n+m_1>4, \quad n+m_2>4$;

Theorem 2. *Let* (2) *and the condition 1 be fulfilled. Suppose that conditions **II** a) or **II** b) are valid. Then the assertion of Theorem 1 holds.*

2. Proof of the theorem on local solvability

In a Hilbert space H define a linear operator Λ as follows

$$\Lambda=\begin{pmatrix} 0 & I & 0 & 0\\ -\Delta_{I_1}^2+\Delta_{J_1} & 0 & 0 & 0\\ 0 & 0 & 0 & I\\ 0 & 0 & -\Delta_{I_2}^2+\Delta_{J_2} & 0\end{pmatrix}, D(\Lambda)=W_{2,1}^{4,2}\times W_{2,1}^{2,1}\times W_{2,2}^{4,2}\times W_{2,2}^{2,1}.$$

Let us also define the nonlinear operator

$$F(t,w)=\begin{pmatrix} 0\\ f_1(t,.,v_1,v_3)\\ 0\\ f_1(t,.,v_1,v_3)\end{pmatrix}, \text{ where } w=\begin{pmatrix} v_1\\ v_2\\ v_3\\ v_4\end{pmatrix}$$

In a Hilbert space H the problem (1), (2) can be written as an operator differential equation:

$$\left.\begin{aligned} w'(t)=\Lambda w(t)+F(t,w(t)),\\ w(0)=w_0, \end{aligned}\right\} \tag{12}$$

Let us provide a well-known theorem on the local solvability of the Cauchy problem in a Hilbert space H.

Theorem 3. *(see [4]) Let a linear operator Λ generate a strongly continuous contracting semigroup in the space* H, *the mapping* $(t,w)\to F(t,w):[0,T]\times \mathrm{H}\to \mathrm{H}$ *satisfies the local Lipschitz condition, i.e. for any* t_1, $t_2\in[0,T]$, w^1, $w^2\in \mathrm{H}$

$$\left\|F\left(t_1,w^1\right)-F\left(t_2,w^2\right)\right\|_{\mathrm{H}}\le c\left(\left\|w^1\right\|_{\mathrm{H}},\left\|w^2\right\|_{\mathrm{H}}\right)\cdot\left\|w^1-w^2\right\|_{\mathrm{H}},$$

where $c(\cdot,\cdot)\in C\left(R_+^2\right)$.

Then for any $w_0 \in \mathrm{H}$ *there exists* $T' > 0$ *such that the problem* (1), (3)-(6) *has a unique solution* $w \in C([0,T'];H)$. *Moreover, if* $T_{\max}$ *the length of the maximal interval of the existence of a weak solution is* $w \in C([0,\, T_{\max});H)$, *then one of the following statements is true:*
i) or $T_{\max} = +\infty$,
ii) or $\lim\limits_{t\to T_{\max}-0} \|w(t)\|_H = +\infty$.
If additionally $w_0 \in D(\Lambda)$, *then* $w \in C([0,T'];D(\Lambda)) \bigcap C^1([0,T'];H)$.

Using the definition of the space H and the operator Λ, we obtain the following statement.

Lemma 1. *Linear operator* Λ *generates a strongly continuous contracting semigroup in the space* H.

The following lemma is also true.

Lemma 2. *The mapping* $F(t,w)$ *is a bounded operator acting from* $[0,T]\times \mathrm{H}$ *to* H. *The mapping* $(t,w) \to F(t,w) : [0,T]\times \mathrm{H} \to \mathrm{H}$ *satisfies the local Lipschitz condition, i.e. for any* $t_1,\ t_2 \in [0,T]$, $w^1,\ w^2 \in \mathrm{H}$

$$\left\|F\left(t_1,w^1\right) - F\left(t_2,w^2\right)\right\|_{\mathrm{H}} \le c\left(\left\|w^1\right\|_{\mathrm{H}}, \left\|w^2\right\|_{\mathrm{H}}\right)\cdot \left\|w^1 - w^2\right\|_{\mathrm{H}},$$

where $c(\cdot,\cdot) \in C\left(R_+^2\right)$.

Proof. We will carry out the proof in the case when $n + m_k > 4,\ \ k = 1,2$. The remaining cases are considered analogously.

Using the expression for $F(t,w)$, we obtain that

$$\left\|F\left(t_1,w^1\right) - F\left(t_2,w^2\right)\right\|_{\mathrm{H}}^2 \le$$

$$\le 2\sum_{k=1}^{2}\left\|f_k\left(t_1,x,v_1^1,v_3^1\right) - f_k\left(t_2,x,v_1^2,v_3^2\right)\right\|_{\mathrm{H}}^2 \le G_{1k} + G_{2k} + G_{3k},$$

where

$$G_{1k} = c\int_\Pi \left|\int_0^1 f'_{kt}\left(t_1+\tau(t_2-t_1),x,v_1^1+\tau(v_1^2-v_1^1),v_3^1+(v_2^2-v_2^1)\right)\right|^2 d\tau\, |t_2-t_1|^2\, dx,$$

$$G_{2k} = c\int_\Pi \left|\int_0^1 f'_{kv_1}\left(t_1+\tau(t_2-t_1),x,v_1^1+\tau(v_1^2-v_1^1),v_3^1+(v_2^2-v_2^1)\right)\right|^2 d\tau\, \left|v_1^2-v_1^1\right|^2 dx,$$

$$G_{3k} = c\int_\Pi \left|\int_0^1 f'_{kv_2}\left(t_1+\tau(t_2-t_1),x,v_1^1+\tau(v_1^2-v_1^1),v_3^1+(v_2^2-v_2^1)\right)\right|^2 d\tau\, \left|v_2^2-v_2^1\right|^2 dx.$$

Using the condition **II** b_3) we have

$$G_{1k} \leq c\int_{\Pi}\int_0^1 \left[g_k(x) + \left|\tau v_1^1 + (1-\tau)v_1^2\right|^{p_{k1}} + \left|\tau v_2^1 + (1-\tau)v_2^2\right|^{p_{k2}}\right]^2 d\tau dx \left|t_2 - t_1\right|^2 \leq$$

$$\leq \left[c_1 \int_{\Pi} g_k^2(x)dx + c_1 \int_0^1 \int_{\Pi} \left|\tau v_1^1 + (1-\tau)v_1^2\right|^{2p_{k1}} dx + \right.$$

$$\left. + c_1 \int_0^1 \int_{\Pi} \left|\tau v_2^1 + (1-\tau)v_2^2\right|^{2p_{k2}} dx\right] . \left|t_2 - t_1\right|^2 \qquad (13)$$

Taking into account the expression $W_{2,k}^{2,1} \subset L_{\frac{n+m_k}{n+m_k-4}}(\Pi)$, $k := 1,2$ (see [3]) from (13) we obtain that

$$G_{1k} \leq C\left[1 + \int_0^1 \left\|\tau v_1^1 + (1-\tau)v_1^2\right\|_{W_{2,k}^{2,1}}^{2p_{k1}} d\tau + \int_0^1 \left\|\tau v_2^1 + (1-\tau)v_2^2\right\|_{L_2(\Pi)}^{2p_{k2}} d\tau\right] . \left|t_2 - t_1\right|^2 \leq$$
$$\leq C\left[1 + \left\|v_1^1\right\|_{W_{2,1}^{2,1}}^{2p_{k1}} + \left\|v_1^2\right\|_{W_{2,1}^{2,1}}^{2p_{k1}} + \left\|v_2^1\right\|_{W_{2,2}^{2,1}}^{2p_{k2}} + \left\|v_2^2\right\|_{W_{2,2}^{2,1}}^{2p_{k2}}\right] . \left|t_2 - t_1\right|^2 \quad . \qquad (14)$$

Similarly using the condition **II** $b_3)$ we have

$$G_{2k} \leq c\int_{\Pi}\int_0^1 \left[h_k(x) + \left|\tau v_1^1 + (1-\tau)v_1^2\right|^{q_{k1}} + \left|\tau v_2^1 + (1-\tau)v_2^2\right|^{q_{k2}}\right]^2 \left|v_1^2 - v_1^1\right|^2 d\tau dx \leq$$

$$\leq \left[c_1 \left(\int_{\Pi} h_k^{2\alpha_k'}(x)dx\right)^{1/\alpha_k'} + \left(\int_{\Pi} \left|\tau v_1^1 + (1-\tau)v_1^2\right|^{2q_{k1}\alpha_k'} dx\right)^{1/\alpha_k'} + \right.$$

$$\left. + \left(\int_{\Pi} \left|\tau v_2^1 + (1-\tau)v_2^2\right|^{2q_{k2}\alpha_k'}(x)dx\right)^{1/\alpha_k'}\right] . \left[\int_{\Pi} \left|v_1^2 - v_1^1\right|^{2\alpha_k} dx\right]^{1/\alpha_k}, \qquad (15)$$

where $\alpha_k = \frac{n+m_1}{n+m_1-4}$, $\alpha_k' = \frac{n+m_1}{4}$.

By virtue of (10) $2q_{k1}\alpha_k' \leq \frac{2(n+m_1)}{n+m_1-4}$, $2q_{k2}\alpha_k' \leq \frac{2(n+m_2)}{n+m_2-4}$, $k = 1,2$, and thus

$$\left[\int_{\Pi} \left|v_1^2 - v_1^1\right|^{2\alpha_k} dx\right]^{1/\alpha_k} \leq c\left\|v_1^2 - v_1^1\right\|_{W_{2,1}^{2,1}}^2, \qquad (16)$$

$$\left(\int_{\Pi} \left|\tau v_1^1 + (1-\tau)v_1^2\right|^{2q_{k1}\alpha_k'} dx\right)^{1/\alpha_k'} \leq$$

$$\leq c\left\|\tau v_1^1 + (1-\tau)v_1^2\right\|_{W_{2,1}^{2,1}}^{2q_{k1}} \leq C\left(\left\|v_1^1\right\|_{W_{2,1}^{2,1}}^2 + \left\|v_1^2\right\|_{W_{2,1}^{2,1}}^2\right)^{q_{k1}}, \qquad (17)$$

$$\left(\int_{\Pi} \left|\tau v_2^1 + (1-\tau)v_2^2\right|^{2q_{k2}\alpha_k'} dx\right)^{1/\alpha_k'} \leq$$

$$\leq c\left\|\tau v_2^1+(1-\tau)v_2^2\right\|_{W_{2,2}^{2,1}}^{2q_{k2}} \leq C\left(\left\|v_2^1\right\|_{W_{2,2}^{2,1}}^2+\left\|v_2^2\right\|_{W_{2,2}^{2,1}}^2\right)^{q_{k2}}. \tag{18}$$

From (14)-(18) it follows

$$G_{2k} \leq C\left[1+\left\|v_1^1\right\|_{W_{2,1}^{2,1}}^{2q_{k1}}+\left\|v_1^2\right\|_{W_{2,1}^{2,1}}^{2q_{k1}}+\left\|v_2^1\right\|_{W_{2,2}^{2,1}}^{2q_{k2}}+\left\|v_2^2\right\|_{W_{2,2}^{2,1}}^{2q_{k2}}\right]\cdot\left\|v_1^2-v_1^1\right\|_{W_{2,1}^{2,1}}^2. \tag{19}$$

Taking into account (9)-(11) we similarly obtain

$$G_{3k} \leq C\left[1+\left\|v_1^1\right\|_{W_{2,1}^{2,1}}^{2r_{k1}}+\left\|v_1^2\right\|_{W_{2,1}^{2,1}}^{2r_{k1}}+\left\|v_2^1\right\|_{W_{2,2}^{2,1}}^{2r_{k2}}+\left\|v_2^2\right\|_{W_{2,2}^{2,1}}^{2r_{k2}}\right]\cdot\left\|v_1^2-v_1^1\right\|_{W_{2,1}^{2,1}}^2. \tag{20}$$

Thus, in view of (14), (19),(20) we obtain that

$$\left\|F\left(t_1,w^1\right)-F\left(t_2,w^2\right)\right\|_{\mathrm{H}} \leq c(\left\|w^1\right\|_{\mathrm{H}},\ \left\|w^2\right\|_{\mathrm{H}})\left\|w^2-w^1\right\|_{\mathrm{H}}$$

From Lemmas 1 and 2 it follows that for the problem (2.12) all the conditions of Theorem 3, on local solvability are satisfied.

3. Solvability "as a whole" of the mixed problem for systems of semilinear hyperbolic equations of the fourth order

In the domain $Q_T=[0,T]\times\Pi$ let us consider the mixed problem

$$\left.\begin{array}{l} u_{1tt}+\Delta_{I_1}^2 u_1-\Delta_{J_1}u_1+|u_1|^{p_1-1}\ |u_2|^{p_2+1}\ u_1=f_1\left(t,x\right), \\ u_{2tt}+\Delta_{I_2}^2 u_2-\Delta_{J_2}u_2+|u_1|^{p_1+1}\ |u_2|^{p_2-1}\ u_2=f_2\left(t,x\right), \end{array}\right\} \tag{21}$$

with the boundary conditions

$$u_k\left(t,x_1,x_2,...,x_i,...,x_n\right)=0, \qquad x_i=0\ ,x_i=1,k=1,2; \quad i=0,1,...,n; \tag{22}$$

$$\Delta_{I_k}u_k\left(t,x_1,x_2,...,x_i,...,x_n\right)=0, \qquad x_i=0\ ,x_i=1,i\in I_k, \quad k=1,2;\ , \tag{23}$$

and the initial conditions

$$u_k\left(0,x\right)=\varphi_k\left(x\right),\ \ u_{kt}\left(0,x\right)=\psi_k\left(x\right),\ \ x\in\Pi,\ \ k=1,2, \tag{24}$$

where

$$n+m_2\leq 4, \quad p_j\geq 0, j=1,2, \tag{25}$$

$f_k(t,x)$, $k=1,2$ are real functions $t\in[0,\infty)$, $x\in\Pi$ and

$$f_k(.)\ \in\ L_2(0,T;\Pi)\ \ ,\ k=1,2. \tag{26}$$

Theorem 4. *Let all the conditions* (25) *and* (26) *be fulfilled. Then for any* $((\varphi_1,\psi_1),(\varphi_2,\psi_2))\in W_{2,1}^{2,1}\times L_2(R^n)\times W_{2,2}^{2,1}\times L_2(\Pi)$ *and* $T>0$ *the mixed problem* (21)-(24) *has a unique solution.*

$$(u_1,u_2)\in C\left([0,T]\,;W_{2,1}^{2,1}\times W_{2,2}^{2,1}\right)\bigcap C^1\left([0,T]\,;L_2(\Pi)\times L_2(\Pi)\right).$$

Proof. In view of Theorem 1, the problem (1)-(3) has a local solution and for the possibility of a global extension of this solution it is sufficient to perform the following a priori estimation:

$$E(t) = \sum_{k=1}^{2}\left[\int_{\Pi}\left|\Delta_{k} u_{k\tau}(t,\, x)\right|^2 dx + \right.$$

$$\left. + \int_{\Pi}\left|\nabla_{J_k} u_k(t,\, x)\right|^2 dx + \int_{\Pi}\left|u'_{kt}(t,\, x)\right|^2 dx\right] \le C,\ t\in[0,T_{\max}). \quad (27)$$

Let us multiply both sides of the first equation of the system (21) by $(p_1+1)\,u_{1t}(t,x)$, and the second equation by $(p_2+1)\,u_{2t}(t,x)$ and integrate the obtaining identity over the domain $Q_T=[0,t]\times\Pi$:

$$\sum_{k=1}^{2}\frac{p_k+1}{2}\left[\int_{\Pi}\left|u_{k\tau}(t,\, x)\right|^2 dx + \int_{\Pi}\left|\Delta_{k} u_k(t,\, x)\right|^2 dx + \int_{\Pi}\left|\nabla_{J_k} u_k(t,\, x)\right|^2 dx\right] +$$

$$+ \int_{\Pi}\left|u_1(\tau,\, x)\right|^{p_1+1}\left|u_2(\tau,\, x)\right|^{p_2+1} dx d\tau =$$

$$= \sum_{k=1}^{2}\frac{p_k+1}{2}\left[\int_{\Pi}\left|\psi_k(\,x)\right|^2 dx + \int_{\Pi}\left|\Delta_{I_k}\varphi_k(x)\right|^2 dx + \int_{\Pi}\left|\nabla_{J_k}\varphi_k(x)\right|^2 dx\right] +$$

$$+\sum_{k=1}^{2}(p_k+1)\int_0^t\int_{\Pi} f(\tau,x).u_{k\tau}(\tau,\, x)\, dx d\tau\ .$$

Further, applying the Hölder inequality and taking (25), (26) into account, we have:

$$\sum_{k=1}^{2}\frac{p_k+1}{2}\left[\int_{\Pi}\left|u_{k\tau}(t,\, x)\right|^2 dx + \int_{\Pi}\left|\Delta_{k} u_k(t,\, x)\right|^2 dx + \int_{\Pi}\left|\nabla_{J_k} u_k(t,\, x)\right|^2 dx\right] +$$

$$+ \int_{\Pi}\left|u_1(t,\, x)\right|^{p_1+1}\left|u_2(t,\, x)\right|^{p_2+1} dx d\tau \le C + \sum_{k=1}^{2}\frac{p_k+1}{2}\int_0^t\int_{\Pi}\left|u_{k\tau}(\tau,\, x)\right|^2\, dx d\tau,$$

where

$$C = \sum_{k=1}^{2}\frac{p_k+1}{2}\left[\int_{\Pi}\left|\psi_k(\,x)\right|^2 dx + \int_{\Pi}\left|\Delta_{I_k}\varphi_k(x)\right|^2 dx + \int_{\Pi}\left|\nabla_{J_k}\varphi_k(x)\right|^2 dx\right] +$$

$$+\sum_{k=1}^{2}\frac{p_k+1}{2}\int_0^T\int_{\Pi}\left|u_{k\tau}(\tau,\, x)\right|^2\, dx d\tau\ .$$

Here applying Gronwall's lemma (see [5]) we obtain the estimation (27).

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