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About a Problem of Optimal Investment of the Stock Market

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Abstract. The article analyzes the investment portfolio containing 3 assets from the Russian stock market. This analysis provides an opportunity to determine how to make an effective portfolio with less risk. The article takes into account the real data on financial instruments. The calculations are presented in Microsoft Excel program.

Key Words and Phrases: correlation, covariance, risk, expected return, portfolio variance, standard deviation, effective portfolio, effective frontier, Markowitz model.

2010 Mathematics Subject Classifications: 74P05; 91G10

In this article, the problem of constructing an optimal portfolio of stocks for a given level of return is solved. To solve the problem, two methods were used: the Solver command from the Excel application program and the Lagrange multiplier method. What is more, the first method is considered for both cases, when short sales of assets are allowed and prohibited. The application of the second method is described only for the case of the possibility of short sales, in that the condition that the Lagrange method is not negative is not considered. Comparison of the effectiveness of these products in all cases can be performed by comparing the final results of portfolio management. One of the classic problems of optimization of the portfolio of these securities is the task of determining the relative densities of each of these securities.

Suppose an investor has a portfolio of N kinds of assets. It is necessary to decide what specific weight each of these securities should have, so that the investor can obtain the desired return, and the portfolio risk at the same time would be reduced to a minimum. As it is known [1], [2], to solve this problem the expected return of each of the securities and their matrix of pairwise covariance will be needed. The whole task is led to minimizing the portfolio variance.

$$\sigma_\rho^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}\theta_i\theta_j cov_{(r_i r_j)}. \tag{1}$$

Under two limiting conditions:

1) the expected return on the portfolio (r_p) is equals to:

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$$\overline{r_p} = \sum_{i=1}^{n} \theta_i \overline{r}_i \tag{2}$$

where - r_i - the profitability of the i-th security, and θ_i - its weight in the portfolio.
2) the sum of the weights of all assets is equal to one:

$$\sum_{i=1}^{n} \theta_i = 1. \tag{3}$$

In this paper, we examine some of the important tasks for the numerical computer calculation of optimal portfolio shares. As we observe the three companies as Aeroflot, Megafon, and Rosneft for 2016. The goal of the task is to find the optimal proportion of these shares using the method of Harry Markowitz.

In order to determine the expected return, we need the real yields of assets that we would like to include in the portfolio. For example, take the data in a percentage of Aeroflot, Megafon, and Rosneft for 2016. The investment period equals to one month in 2016.

Using formulas (1), (2) and (3), we will compile a model for this portfolio of three stocks. Find $\theta = (\theta_1, \theta_2, \theta_3)$:

$$\sigma_\rho^2 = \theta_1^2\sigma_1^2 + \theta_2^2\sigma_2^2 + \theta_3^2\sigma_3^2 + 2\theta_1\theta_2 cov_{1,2} + 2\theta_1\theta_3 cov_{1,3} + 2\theta_2\theta_3 cov_{2,3} \;\to min \tag{4}$$

$$\begin{cases} \theta_1\overline{r_1} + \theta_2\overline{r_2} + \theta_3\overline{r_3} = \overline{r_p} \\ \sum_i^3 \theta_i = 1 \\ \theta_i \geq 0.1 \quad i = \overline{1,3} \end{cases}$$

	A	B	C	D
1	**month**	**Aeroflot**	**Megafon**	**Rosneft**
2	**January**	-9.43	1.41	7.10
3	**February**	12.18	-0.46	5.64
4	**March**	30.21	-13.79	6.42
5	**April**	4.35	-1.07	15.94
6	**May**	6.04	1.22	-8.84
7	**June**	3.52	-10.06	4.38
8	**July**	0.69	-0.07	-1.17
9	**August**	15.04	-0.37	5.57
10	**September**	18.23	-8.83	-0.57
11	**October**	11.13	-0.82	0.77
12	**November**	3.60	-6.27	-2.31
13	**December**	12.41	1.60	17.38

Table 1. Monthly growth for 2016

For the case of the absence of dividend payments, the yield coefficient can be determined from the formula:

$$r = (P_1 - P_0)/P_0, \tag{5}$$

where P_0 - is the value of the security in the initial period; P_1 - the value of the security at the end of the period. [6]
Knowing the monthly returns, we will try to determine the expected return on each of the assets. To find the expected (average) yield of Aeroflot, Megafon, and Rosneft, we need the following table:

AEROFLOT				MEGAFON				ROSNEFT			
return	frequency	probability		return	frequency	probability		return	frequency	probability	
-9.43	1.00	0.08	-0.79	1.41	1	0.08	0.12	7.10	1	0.08	0.59
12.18	1.00	0.08	1.02	-0.46	1	0.08	-0.04	5.64	1	0.08	0.47
30.21	1.00	0.08	2.52	-13.79	1	0.08	-1.15	6.42	1	0.08	0.53
4.35	1.00	0.08	0.36	-1.07	1	0.08	-0.09	15.94	1	0.08	1.33
6.04	1.00	0.08	0.50	1.22	1	0.08	0.10	-8.84	1	0.08	-0.74
3.52	1.00	0.08	0.29	-10.06	1	0.08	-0.84	4.38	1	0.08	0.37
0.69	1.00	0.08	0.06	-0.07	1	0.08	-0.01	-1.17	1	0.08	-0.10
15.04	1.00	0.08	1.25	-0.37	1	0.08	-0.03	5.57	1	0.08	0.46
18.23	1.00	0.08	1.52	-8.83	1	0.08	-0.74	-0.57	1	0.08	-0.05
11.13	1.00	0.08	0.93	-0.82	1	0.08	-0.07	0.77	1	0.08	0.06
3.60	1.00	0.08	0.30	-6.27	1	0.08	-0.52	-2.31	1	0.08	-0.19
12.41	1.00	0.08	1.03	1.60	1	0.08	0.13	17.38	1	0.08	1.45
Sum	**12.00**	**1.00**	**9.00**	**Sum**	**12**	**1**	**-3.13**	**Sum**	**12**	**1**	**4.19**

Table 2. Expected returns

After determining the expected return values of assets, we can proceed to the calculation of risks, which are determined by dispersion and standard deviations, as a measure of the spread of possible outcomes relative to the expected value. Consequently, the higher the dispersion, the greater the spreading, and hence the risk. To calculate the dispersion, the following formula is chosen:

$$\sigma^2 = \frac{\sum_{i=1}^{n} r_i - r}{(n-1)} \tag{6}$$

r_i – return of the asset, r_{aver}-expected (average) return on an asset, n-number of observations.
To calculate the "standard quadratic deviation" (standard deviation), which is the square root of the dispersion, is used:

$$\sigma_p = \sqrt{\sigma^2} \tag{7}$$

Here is an example of calculation of dispersion and standard deviation using Microsoft Excel based on the available data of Aeroflot, Megafon, and Rosneft, using the built-in functions of **VAR** and **STDEV**.
As a result, we get:

AEROFLOT				MEGAFON				ROSNEFT			
return	frequency	probability		return	frequency	probability		return	frequency	probability	
-9.43	1.00	0.08	-0.79	1.41	1	0.08	0.12	7.10	1	0.08	0.59
12.18	1.00	0.08	1.02	-0.46	1	0.08	-0.04	5.64	1	0.08	0.47
30.21	1.00	0.08	2.52	-13.79	1	0.08	-1.15	6.42	1	0.08	0.53
4.35	1.00	0.08	0.36	-1.07	1	0.08	-0.09	15.94	1	0.08	1.33
6.04	1.00	0.08	0.50	1.22	1	0.08	0.10	-8.84	1	0.08	-0.74
3.52	1.00	0.08	0.29	-10.06	1	0.08	-0.84	4.38	1	0.08	0.37
0.69	1.00	0.08	0.06	-0.07	1	0.08	-0.01	-1.17	1	0.08	-0.10
15.04	1.00	0.08	1.25	-0.37	1	0.08	-0.03	5.57	1	0.08	0.46
18.23	1.00	0.08	1.52	-8.83	1	0.08	-0.74	-0.57	1	0.08	-0.05
11.13	1.00	0.08	0.93	-0.82	1	0.08	-0.07	0.77	1	0.08	0.06
3.60	1.00	0.08	0.30	-6.27	1	0.08	-0.52	-2.31	1	0.08	-0.19
12.41	1.00	0.08	1.03	1.60	1	0.08	0.13	17.38	1	0.08	1.45
Sum	**12.00**	**1.00**	**9.00**	**Sum**	**12**	**1**	**-3.13**	**Sum**	**12**	**1**	**4.19**
Dispersion		**99.66**		**27.23**				**54.97**			
Standard deviation		**9.98**		**5.22**				**7.41**			

Table 3.Dispersions and standard deviations

Final calculations allow us to state that the least risky paper is Megafon. The expected monthly return is -3.13% at a risk of 5.22%. And the riskiest paper is Aeroflot. It should be noted that not always the asset having the highest standard deviation is the riskiest. Therefore, before using the standard deviation as a measure of relative risk, you need to calculate the risk per unit of return using the coefficient of variation. This indicator will not be analyzed, as applied to the formation of our portfolio, it is not necessary.

Knowing the expected returns and risk indicators (standard deviation), it is necessary to make a number of calculations to determine the coefficients of covariance and correlation. After calculating these coefficients, it becomes possible to form portfolios that meet our requirements for risk and return. Note that covariance is a measure that takes into account the dispersion of individual values of the return of a paper and the strength of the links between changes in the returns of this paper and others.

The formula for calculating the covariance is the following:

$$cov\left(\theta_i \theta_j\right) = \frac{\sum_{i=1}^{n}\left(r_{\theta_i} - r_{\theta_i}\right) * \left(r_{\theta_j} - r_{\theta_j}\right)}{n-1} \tag{8}$$

where r_X and r_Y – returns of assets X and Y, rx_{aver} and rY_{aver}– expected (average) returns of the assets X and Y, n – number of observations.

The positive value of covariance indicates that the values of the returns of these assets change in one direction, the negative value of covariance indicates multidirectional movements between returns. Covariance is low if the fluctuations in the returns of the two assets in any direction are random. To interpret covariance, as well as dispersion, is quite difficult due to large numerical values, so almost always a correlation coefficient is used to measure the strength of the relationship between the two assets.

The correlation coefficient lies in the interval from -1 to +1. A correlation value of +1 indicates a strong relationship, as assets go the same way. The value -1, on the contrary, indicates a different direction, to wit, the growth of one of the assets is accompanied by the fall of the other. A value of 0 indicates no correlation.
To calculate the correlation in the EXCEL environment, we will use the following formula:

$$\rho = \frac{cov(\theta_i, \theta_j)}{\sigma_{\theta_i} * \sigma_{\theta_j}} \tag{9}$$

where $cov(X, Y)$ – covariance between to assets X and Y in the denominator are standard deviations of the assets X and Y.
Here are the calculations of covariance and correlation using Excel between Aeroflot, Megafon, and Rosneft, using the built-in **COVAR** and **CORREL** functions.

month	Aeroflot	Megafon	Rosneft
January	-9.43	1.41	7.10
February	12.18	-0.46	5.64
March	30.21	-13.79	6.42
April	4.35	-1.07	15.94
May	6.04	1.22	-8.84
June	3.52	-10.06	4.38
July	0.69	-0.07	-1.17
August	15.04	-0.37	5.57
September	18.23	-8.83	-0.57
October	11.13	-0.82	0.77
November	3.60	-6.27	-2.31
December	12.41	1.60	17.38

Covariance			
	Aeroflot	Megafon	Rosneft
Aeroflot	91.36	-25.76	5.26
Megafon	-25.76	24.96	4.08
Rosneft	5.26	4.08	50.39

Correlation			
	Aeroflot	Megafon	Rosneft
Aeroflot	1.00	-0.54	0.08
Megafon	-0.54	1.00	0.11
Rosneft	0.08	0.11	1.00

Table 4. Covariance and Correlations

As can be seen from the correlation table, the monthly returns of our assets in the segment of 2015 are not quite positively correlated, which, of course, is not very good, but even the inclusion of positively correlated assets in the portfolio can significantly reduce the risk of the entire portfolio. Having all the data that you can now transfer to the formation of the portfolio and the problems associated with it.

An investment portfolio - is a purposefully formed set of objects of real and financial investment for the implementation of the investment policy of the enterprise in the forthcoming period. The main objective in the formation of the investment portfolio is to ensure the implementation of the main areas of the investment activity of the enterprise by selecting the most profitable and safe investment objects. Taking into consideration the main goal, a system of specific local goals is being constructed, the main of which are:

1. high level of income in the current period

2. minimization of investment risks

3. sufficient liquidity of the investment portfolio

These specific objectives of forming an investment portfolio are largely alternative. Thus, ensuring a high growth rate of capital, in the long run, is achieved to a certain extent by reducing the current return level of the investment portfolio, and vice versa. The level of the current return on the investment portfolio directly depends on the level of investment risks. Ensuring sufficient liquidity can prevent the inclusion of investment projects in the portfolio aimed at obtaining high capital growth in the long-term. Given the alternatives of the objectives of forming an investment portfolio, each investor himself sets their priorities.

The alternative nature of the objectives of forming an investment portfolio determines the differences in the company's financial investment policy, which in turn provides a specifically formed type of investment portfolio. The income portfolio is formed by the criterion of maximizing the level of investment profit in the current period, regardless of the growth rate of the invested capital in the long term. In other words, this portfolio is focused on high current return on investment costs, despite the fact that in the future period these costs could provide a higher rate of investment return on invested capital. [7]

Now, let's begin to search for the optimal portfolio.

According to Markowitz, any investor should base his choice exclusively on the expected return and the standard deviation when choosing a portfolio. Thus, having assessed various combinations of portfolios, he should choose the "best", based on the ratio of expected returns and standard deviation of these portfolios. At the same time, the ratio of return on portfolio risk remains the same: the higher the return, the higher the risk.

Also, before starting to form a portfolio, it is necessary to define the term "effective portfolio". An effective portfolio is a portfolio that provides: the maximum expected return for a certain level of risk, or the minimum level of risk for some expected return.

In the future, we will find efficient portfolios in the Excel environment in accordance with the second principle - with a minimum level of risk for any expected return. In order to find the optimal portfolio, it is necessary to determine the acceptable set of risk-return ratios for the investor, which is achieved by constructing a minimal dispersion portfolio frontier, that is, the border on which portfolios with a minimum risk for a given return lies.

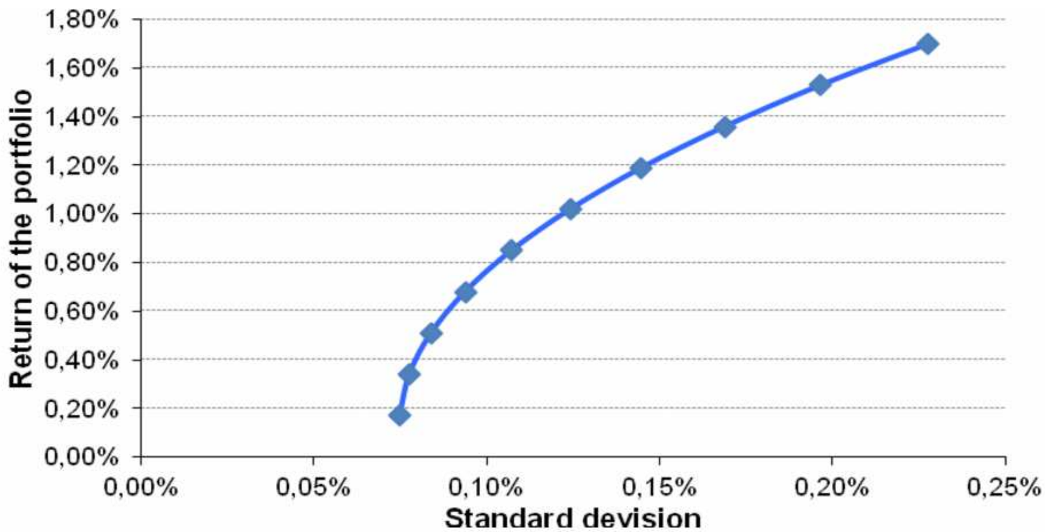

Table 5.Effective frontier

In the table above, the bold line shows the "effective frontier", and the big points indicate possible combinations of portfolios.

An effective frontier - is a boundary that defines an effective set of portfolios. Portfolios lying to the left of the effective boundary can not be applied, that is, they do not belong to an admissible set. Portfolios on the right (internal portfolios) and below the effective border are ineffective because there are portfolios that at a given level of risk provide higher returns or lower risk for a given level of profitability.
Let's start by calculating the expected return on the portfolio using the formula:

$$E\left(r_{\rho}\right)=\sum_{i=1}^{n} \theta_{i} E\left(r_{i}\right), \tag{10}$$

where X_i – the share of the i-th paper in the portfolio:
$E(r_i)$ – the expected return on the i-th paper:
And then determine the dispersion of the portfolio, in the formula which uses double summation:

$$\sigma_{\rho}^{2}=\sum_{i=1}^{n} \sum_{j=1}^{n} \theta_{i} \theta_{j} cov_{\left(r_{i} r_{j}\right)}, \tag{11}$$

where $\sigma\ \rho 2$ – dispersion of portfolio:
$X_i X_j$ – the proportion of the i-th and j-th paper in the portfolio:
$cov(ri, rj)$ – covariance of returns of papers i and j.
As a consequence, we find the standard deviation of the portfolio which is the square root of the dispersion of the formula:

$$\sigma = \sqrt{\sigma^2} \tag{12}$$

Applying the above formula, we set the share of each asset in our initial portfolio in proportion to their number. Therefore, the share of each asset in the portfolio will be 1/3, to wit, 33%. The total share should be equal to 1, both for portfolios in which "short" positions are allowed and for those that are prohibited. If "short" positions are allowed, the share of the asset will be displayed as -0.33 and the proceeds from its sale must be invested in another asset, so the share of assets in the portfolio will, in any case, be 1. [8]

We calculate the expected return, dispersion and standard deviation of the average weighted portfolio. As can be seen from the table, in order to determine the portfolio dispersion, you just need to sum the data in cells B28-D28, and the square root of the cell value C31 will give us the standard deviation of the portfolio in cell C32. The multiplication of shares of papers by their expected return will give us the expected return on our portfolio, which is reflected in cell C33. The final result of the average weighted portfolio is presented below.

	Аэрофлот	Мегафон	Роснефть
Аэрофлот	91.36	-25.76	5.26
Мегафон	-25.76	24.96	4.08
Роснефть	5.26	4.08	50.39

	Аэрофлот	Мегафон	Роснефть
Аэрофлот	1.00	-0.54	0.08
Мегафон	-0.54	1.00	0.11
Роснефть	0.08	0.11	1.00

	Аэрофлот	Мегафон	Роснефть
доля	0.33	0.33	0.33
0.33	10.15	-2.86	0.58
0.33	-2.86	2.77	0.45
0.33	0.58	0.45	5.60
1.00	7.87	0.36	6.64

	Ожидаемая доходность	Стандарт отклонение
Аэрофлот	9.00	9.98
Мегафон	-3.13	5.22
Роснефть	4.19	7.41

Дисперсия портфеля	14.87
Стандарт отклон. Порт	3.86
Ожидаемая доход. Порт.	3.35

Table 6.

The average (expected) monthly return of the average weighted portfolio is 3.35% at a risk of 3.86%. Now we can apply the very second principle, which was written above, that is, to ensure a minimum risk at a given level of profitability. To do this, use the "Find Solutions" function from the "Tools" menu. [9]

We launch the "Find solutions", in the item "Set the specified cell" we specify cell C32, which we will minimize by changing the shares of assets in the portfolio, that is, by varying the values in cells A25-A27. Then we need to add two conditions, namely:
1) the sum of shares must equal 1; cell A28 = 1,

2) To set the profitability that interests us, for example, the return of 3.35% (cell 33), which turned out when calculating the average weighted portfolio.
As we prohibit the presence of "short" positions on securities in the "Options" menu, it is necessary to put a tick in the box "Non-negative values". This should look like this:

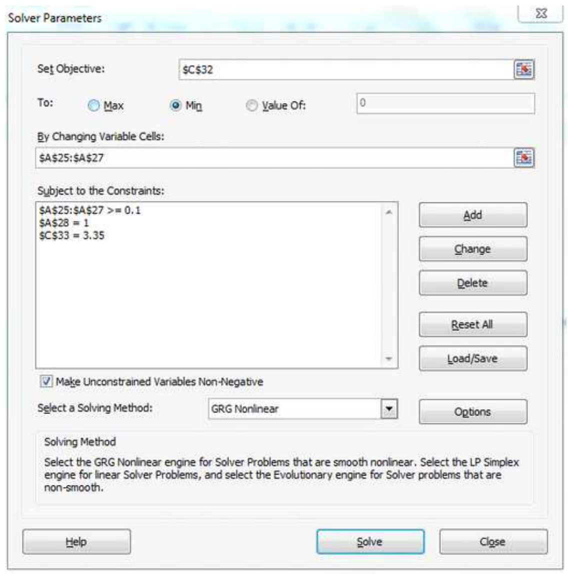

Table 7. "Solver"

Table 8. Solver options

As a result, we get:

Covariance	Aeroflot	Megafon	Rosneft
Aeroflot	91.36	-25.76	5.26
Megafon	-25.76	24.96	4.08
Rosneft	5.26	4.08	50.39

Correlation	Aeroflot	Megafon	Rosneft
Aeroflot	1.00	-0.54	0.08
Megafon	-0.54	1.00	0.11
Rosneft	0.08	0.11	1.00

	Aeroflot	Megafon	Rosneft
shares	0.33	0.33	0.33
0.47	14.43	-4.07	0.83
0.43	-3.66	3.55	0.58
0.10	0.18	0.14	1.68
1.00	**10.95**	**-0.39**	**3.09**

	Expected returns	Standard deviation
Aeroflot	9.00	9.98
Megafon	-3.13	5.22
Rosneft	4.19	7.41

Dispersion of portfolio	13.65
Standard deviation of port.	3.69
Expected return of port.	3.35

Table 9. Result of "Finding Solutions"

So, having set "Finding solutions" to find the minimum standard deviation for a given expected return of 0.33%, we got the optimal portfolio consisting of 47% of Aeroflot, 43% of Megafon and 10% of Rosneft. Despite the fact that the level of profitability is the same as with the average weighted portfolio, the risk has decreased.

Let's observe a number of drawbacks inherent in the model H. Markowitz.

This model was developed for efficient capital markets, where there is a constant growth in the value of assets and there are no sharp fluctuations in rates, which was more typical for the economy of the developed countries of the 50-80s. The correlation between shares is not constant and changes with time, as a result in the future this does not reduce the systematic risk of the investment portfolio.

The future profitability of financial instruments (assets) is determined as the arithmetic mean. This forecast is based only on the historical significance of the stock's returns and does not include the impact of macroeconomic (GDP, inflation, unemployment, sectoral indices of prices for commodities and materials, etc.) and microeconomic factors (liquidity, profitability, financial stability, business activity of the company).

The risk of a financial instrument is estimated using a measure of return variability related to the arithmetic mean, but a change in return above is not a risk but represents a stock's super-returns.

For the case where short sales of assets are allowed, an analytically effective boundary and an effective portfolio can be found using the Lagrange multiplier method. The problem is led to minimizing the dispersion of the portfolio [5] and [10]:

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \theta_i \theta_j cov_{ij} \tag{13}$$

Under two limiting conditions:

1) the expected return on the portfolio (r_p) is:

$$\overline{r_p} = \sum_{i=1}^{n} \theta_i \overline{r}_i \tag{14}$$

2) the sum of the weights of all assets is equal to one:

$$\sum_{i=1}^{n} \theta_i = 1 \tag{15}$$

Artificially created and minimized the Lagrange function in the form:

$$L = G + \lambda_1 C_1 + \lambda_2 C_2 \tag{16}$$

Where L is the Lagrange function;

G - objective function;

$\lambda 1$ and $\lambda 2$ are the Lagrange multipliers for the first and second constraints;

C_1, C_2 - first and second constraints.

The objective function represented by the function (14), first constraints - the equation (15), the second - (16). In the Lagrange function, we include the first, second and third constraints in the following form:

$$\begin{cases} \sum_{i=1}^{3} \theta_i \overline{r}_i - \overline{r_p} = 0 \\ \sum_{i=1}^{3} \theta_i - 1 = 0 \\ \theta_i \geq 0.1 \quad i = 1, 3 \end{cases} \tag{17}$$

The standard deviation of the return of the first asset (in decimal values) is 9.98, the second - 5.22, the third - 7.41. The covariance of the return of the first and second stocks is - (-25.7624), the first and third - 5.2605, the second and the third - 4.0757. The return of the first paper (in decimal values) is 9.00, the second - (-3.13), the third - 4.19. Determine the specific weight of assets in the portfolio with a return of 3.35.

As an objective function to be optimized in this task performs a function:

$$F(\theta) = \theta_1^2 9,98^2 + \theta_2^2 5,22^2 + \theta_3^2 7,41^2 + 2\theta_1\theta_2(-25.76) + 2\theta_1\theta_3 5.26 + 2\theta_2\theta_3 4.08 \tag{18}$$

We rewrite the restriction of the problem in an implicit form:

$$\begin{cases} \varphi_1(\theta) = 3.35 - (9\theta_1 - 3.13\theta_2 + 4.19\theta_3) = 0 \\ \varphi_2(\theta) = 1 - (\theta_1 + \theta_2 + \theta_3) = 0 \\ \varphi_3(\theta) = 0.1 - (\theta_3) = 0 \end{cases} \tag{19}$$

Let us compose the auxiliary Lagrange function. Since the objective function is not linear in the formula. In this case, the Kuhn-Tucker method [12] should be applied in order to solve the problem posed:

$$\mathrm{L}(\theta, \lambda, \mu) = 9,98^2\theta_1^2 + 5.22^2\theta_2^2 + 7.41^2\theta_3^2 + 2\theta_1\theta_2(-25.76) +$$

$$+2\theta_1\theta_3\,(5.26)+2\theta_2\theta_3(4.08)\;+\;\lambda_1(3.35-(9\theta_1-3.13\theta_2+4.19\theta_3))+$$
$$+\lambda_2(1-(\theta_1+\theta_2+\theta_3))\;+\;\mu_3\,(0.1-(\theta_3)) \tag{20}$$

A necessary condition for the extremum of the Lagrange function is the vanishing of its partial derivatives with respect to the variables and θ_i, i = 1,2,3 undetermined factors.

Set up a system:

$$\begin{cases} \frac{\partial L}{\partial \theta_1} = 199,2\theta_1 - 51,52\theta_2 + 10,52\theta_3 - 9\lambda_1 - \lambda_2 = 0 \\ \frac{\partial L}{\partial \theta_2} = -51,52\theta_1 + 54,5\theta_2 + 8,16\theta_3 + 3,13\lambda_1 - \lambda_2 = 0 \\ \frac{\partial L}{\partial \theta_3} = 10,52\theta_1 + 8,16\theta_2 + 109,8\theta_3 - 4,19\lambda_1 - \lambda_2 - \mu_3 = 0 \\ \frac{\partial L}{\partial \lambda_1} = 3,35 - (9\theta_1 - 3,13\theta_2 + 4,19\theta_3) = 0 \\ \frac{\partial L}{\partial \lambda_2} = 1 - (\theta_1 + \theta_2 + \theta_3) = 0 \\ \mu_3\,(0,1 - (\theta_3)) = 0, \qquad \mu_3 \geq 0 \end{cases} \tag{21}$$

Let us solve the following system of equations:

$$\begin{cases} 199,2\theta_1 - 51,52\theta_2 + 10,52\theta_3 - 9\lambda_1 - \lambda_2 = 0 \\ -51,52\theta_1 + 54,5\theta_2 + 8,16\theta_3 + 3,13\lambda_1 - \lambda_2 = 0 \\ 10,52\theta_1 + 8,16\theta_2 + 109,8\theta_3 - 4,19\lambda_1 - \lambda_2 - \mu_3 = 03,35 - (9\theta_1 - 3,13\theta_2 + 4,19\theta_3) = 0 \\ 1 - (\theta_1 + \theta_2 + \theta_3) = 0 \\ \mu_3\,(0,1 - (\theta_3)) = 0, \qquad \mu_3 \geq 0 \end{cases} \tag{22}$$

Solving the system of equations (22) by the inverse matrix method we finally obtain:
θ_1= 0,4739 or 47.39 %
θ_2= 0,4261 or 42.61 %
θ_3= 0,10 or 10 %

Similarly, we set restrictions with a weight of 10% for the second and first shares in the portfolio, and find a solution using the Kuhn-Tucker method:

$$\mathrm{L}(\theta,\,\lambda,\,\mu)\;=9,98^2\theta_1^2{+}5.22^2\theta_2^2{+}7.41^2\theta_3^2{+}2\theta_1\theta_2(-25.76)+2\theta_1\theta_3\,(5.26)+$$
$$+2\theta_2\theta_3(4.08)\;+\;\lambda_1(3.35-(9\theta_1-3.13\theta_2+4.19\theta_3))\;+\;\lambda_2(1-(\theta_1+\theta_2+\theta_3))\;+\;\mu_3\,(0.1-(\theta_2)) \tag{23}$$

Let's make the system:

$$\begin{cases} \frac{\partial L}{\partial \theta_1} = 199,2\theta_1 - 51,52\theta_2 + 10,52\theta_3 - 9\lambda_1 - \lambda_2 = 0 \\ \frac{\partial L}{\partial \theta_2} = -51,52\theta_1 + 54,5\theta_2 + 8,16\theta_3 + 3,13\lambda_1 - \lambda_2 = 0 \\ \frac{\partial L}{\partial \theta_3} = 10,52\theta_1 + 8,16\theta_2 + 109,8\theta_3 - 4,19\lambda_1 - \lambda_2 - \mu_3 = 0 \\ \frac{\partial L}{\partial \lambda_1} = 3,35 - (9\theta_1 - 3,13\theta_2 + 4,19\theta_3) = 0 \\ \frac{\partial L}{\partial \lambda_2} = 1 - (\theta_1 + \theta_2 + \theta_3) = 0 \\ \mu_3\,(0,1 - (\theta_2)) = 0, \qquad \mu_3 \geq 0 \end{cases} \tag{24}$$

Let us solve the following system of equations:

$$\begin{cases} 199,2\theta_1 - 51,52\theta_2 + 10,52\theta_3 - 9\lambda_1 - \lambda_2 = 0 \\ -51,52\theta_1 + 54,5\theta_2 + 8,16\theta_3 + 3,13\lambda_1 - \lambda_2 = 0 \\ 10,52\theta_1 + 8,16\theta_2 + 109,8\theta_3 - 4,19\lambda_1 - \lambda_2 - \mu_3 = 0 \\ 3,35 - (9\theta_1 - 3,13\theta_2 + 4,19\theta_3) = 0 \\ 1 - (\theta_1 + \theta_2 + \theta_3) = 0 \\ \mu_3\,(0,1 - (\theta_2)) = 0, \qquad \mu_3 \geq 0 \end{cases} \tag{25}$$

Solving the system of equations (25) by the inverse matrix method we finally obtain:
θ_1= - 0, 0225 or -2.25 %
θ_2= 0,1 or 10 %
θ_3= 0,9225 or 92,25 %

$$\mathrm{L}(\theta,\ \lambda,\ \mu)\ = 9,98^2\theta_1^2{+}5.22^2\theta_2^2{+}7.41^2\theta_3^2{+}2\theta_1\theta_2(-25.76) + 2\theta_1\theta_3\,(5.26) +$$

$$+2\theta_2\theta_3(4.08)\ +\ \lambda_1(3.35-(9\theta_1{-}3.13\theta_2{+}4.19\theta_3))\ +\ \lambda_2(1-(\theta_1{+}\theta_2{+}\theta_3))\ +\ \mu_3\,(0.1{-}\,(\theta_1)) \tag{26}$$

Let's make the system:

$$\begin{cases} \frac{\partial L}{\partial \theta_1} = 199,2\theta_1 - 51,52\theta_2 + 10,52\theta_3 - 9\lambda_1 - \lambda_2 = 0 \\ \frac{\partial L}{\partial \theta_2} = -51,52\theta_1 + 54,5\theta_2 + 8,16\theta_3 + 3,13\lambda_1 - \lambda_2 = 0 \\ \frac{\partial L}{\partial \theta_3} = 10,52\theta_1 + 8,16\theta_2 + 109,8\theta_3 - 4,19\lambda_1 - \lambda_2 - \mu_3 = 0 \\ \frac{\partial L}{\partial \lambda_1} = 3,35 - (9\theta_1 - 3,13\theta_2 + 4,19\theta_3) = 0 \\ \frac{\partial L}{\partial \lambda_2} = 1 - (\theta_1 + \theta_2 + \theta_3) = 0 \\ \mu_3\,(0,1 - (\theta_1)) = 0, \qquad \mu_3 \geq 0 \end{cases} \tag{27}$$

Let us solve the following system of equations:

$$\begin{cases} 199,2\theta_1 - 51,52\theta_2 + 10,52\theta_3 - 9\lambda_1 - \lambda_2 = 0 \\ -51,52\theta_1 + 54,5\theta_2 + 8,16\theta_3 + 3,13\lambda_1 - \lambda_2 = 0 \\ 10,52\theta_1 + 8,16\theta_2 + 109,8\theta_3 - 4,19\lambda_1 - \lambda_2 - \mu_3 = 0 \\ 3,35 - (9\theta_1 - 3,13\theta_2 + 4,19\theta_3) = 0 \\ 1 - (\theta_1 + \theta_2 + \theta_3) = 0 \\ \mu_3\,(0,1 - (\theta_1)) = 0, \qquad \mu_3 \geq 0 \end{cases} \tag{28}$$

Solving the system of equations (28) by the inverse matrix method we finally obtain:
θ_1= 0,1 or 10 %
θ_2= 0,1805 or 18,05 %
θ_3= 0,7195 or 71,95 %

Comparing the first, second and third decisions, we see that for an investor with a rational strategy it is important to make such a decision that with a minimum risk to obtain an allowable amount of profit. However, for an investor with a conservative plan of action, it is most likely that it is essential to choose such a combination of risk and income ratios that the maximum profitability will be the priority.

Summary

The studied stock packages in the article are owned by leading Russian stock market companies in real time. The optimization of the investment portfolio here has been resolved both with the Microsoft Excel software package technologically and with the Kunn-Takker method by adding Lagrangian multipliers. Optimization problems by changing restrictive conditions have been investigated, the dynamic structure of the optimal investment portfolio for aggressive investors has been identified.

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