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Some Properties of Two-dimensional Szasz Operator

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Abstract. In the paper, we study some properties of two-dimensional Szasz operator and weighted modulus of continuity on $\Big[0,\infty\Big)$.

Key Words and Phrases: linear positive operator, Szasz operator, weighted modulus of continuity.
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1. Introduction

The polynomial constructed by Bernstein in 1912 for a continuous function has the form

$$B_n\Big(f;x\Big)=\sum_{k=0}^{n} f\Big(\frac{k}{n}\Big)C_n^k x^k\Big(1-x\Big)^{n-k}, 0\le x\le 1, k=0,1,...n.$$

In 1950, Otto Szasz has given a generalization of Bernstein polynomials on the semiaxis $\Big[0,\infty\Big)$ in the following form

$$S_n\Big(f;x\Big)=e^{-nx}\sum_{k=0}^{\infty} f\Big(\frac{k}{n}\Big)\frac{\Big(nx\Big)^k}{k!}, 0\le x<\infty.$$

Note that well known generalization of Bernstein polynomials is given in [6]. But in [7] was given the generalization of Szasz operator as

$$S_{n,r}\Big(f;x\Big)=e^{-nx}\sum_{k=0}^{\infty}\sum_{i=0}^{r}\frac{f^{(i)}\Big(\frac{k}{n}\Big)}{i!}\Big(x-\frac{k}{n}\Big)^i\frac{\Big(nx\Big)^k}{k!},\ \ 0\le x<\infty.$$

Following [8], the generalized operator Szasz order (n,m,r) defined as a

$$S_{n,m,r}(f;x,y)=e^{-nx-my}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{i=0}^{r}\frac{1}{i!}\Big[\Big(x-\frac{k}{n}\Big)\frac{\partial}{\partial x}+\Big(y-\frac{l}{m}\Big)\frac{\partial}{\partial y}\Big]^i f\Big(\frac{k}{n},\frac{l}{m}\Big)\frac{\Big(nx\Big)^k}{k!}\frac{\Big(my\Big)^l}{l!}. \tag{1}$$

In particular, if $r=0$, then (1) coincides with the two dimensional classical operator Szasz

$$S_{n,m}(f;x,y)=e^{-nx}e^{-my}\sum_{k,l=0}^{\infty}f\Big(\frac{k}{n},\frac{l}{m}\Big)\frac{\Big(nx\Big)^{k}}{k!}\frac{\Big(my\Big)^{l}}{l!}. \tag{2}$$

2. Preliminaries and auxiliary results

Lemma 1. *Let f that a continuous function on R_+^2 and $S_{n,m}\Big(f;x,y\Big)$ is Szasz operator (2) of function f. Then the following properties*

$$S_{n,m}(1;x,y)=1,$$
$$S_{n,m}(t;x,y)=x,$$
$$S_{n,m}(\tau;x,y)=y,$$
$$S_{n,m}(t^2;x,y)=x^2+\frac{1}{n}x,$$
$$S_{n,m}(\tau^2;x,y)=y^2+\frac{1}{n}y$$

hold.

Proof. We have

$$S_{n,m}(1;x,y)=e^{-nx}e^{-my}\sum_{k,l=0}^{\infty}\frac{\Big(nx\Big)^{k}}{k!}\frac{\Big(my\Big)^{l}}{l!}=1,\ \ e^{nx}=\sum_{k=0}^{\infty}\frac{\Big(nx\Big)^{k}}{k!},$$

$$S_{n,m}(t;x,y)=e^{-nx}e^{-my}\sum_{k,l=0}^{\infty}\frac{k}{n}\frac{\Big(nx\Big)^{k}}{k!}\frac{\Big(my\Big)^{l}}{l!}=\frac{e^{-nx}}{n}\sum_{k=1}^{\infty}\frac{\Big(nx\Big)^{k}}{\Big(k-1\Big)!}=x,\ \ t=\frac{k}{n},$$

$$S_{n,m}(\tau;x,y)=e^{-nx}e^{-my}\sum_{k,l=0}^{\infty}\frac{l}{m}\frac{\Big(nx\Big)^{k}}{k!}\frac{\Big(my\Big)^{l}}{l!}=\frac{e^{-nx}}{m}\sum_{k=1}^{\infty}\frac{\Big(my\Big)^{l}}{\Big(l-1\Big)!}=y,\ \tau=\frac{l}{m},$$

$$S_{n,m}(t^2;x,y)=e^{-nx}e^{-my}\sum_{k,l=0}^{\infty}\frac{k^2}{n^2}\frac{\Big(nx\Big)^{k}}{k!}\frac{\Big(my\Big)^{l}}{l!}=\frac{e^{-nx}}{n^2}\sum_{k=1}^{\infty}k\frac{\Big(nx\Big)^{k}}{\Big(k-1\Big)!}=x^2+\frac{1}{n}x,$$

$$S_{n,m}(\tau^2;x,y)=e^{-nx}e^{-my}\sum_{k,l=0}^{\infty}\frac{l^2}{m^2}\frac{\Big(nx\Big)^{k}}{k!}\frac{\Big(my\Big)^{l}}{l!}=\frac{e^{-nx}}{m^2}\sum_{k=1}^{\infty}l\frac{\Big(my\Big)^{l}}{\Big(l-1\Big)!}=y^2+\frac{1}{m}y.$$

This complete the proof Lemma 1.

Remark 1. *Note that for Bernstein-Chlodowsky polynomials analog of Lemma 1 was proved in [3]. Also, certain construction of Bernstein-Chlodowsky polynomials of two variable function, corresponding to the certain triangular domain with mobile boundary was given in [3].*

Lemma 2. *For the operator* (1) *the following properties*

$$S_{n,m,r}(1;x,y) = 1,$$
$$S_{n,m,r}(t;x,y) = x,$$
$$S_{n,m,r}(\tau;x,y) = y$$

hold.

Proof. We have

$$S_{n,m,r}(f;x,y) = S_{n,m,r}(1;x,y) =$$
$$= e^{-nx-my} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \sum_{i=0}^{r} \tfrac{1}{i!} \Big[\Big(x - \tfrac{k}{n}\Big)\tfrac{\partial}{\partial x} + \Big(y - \tfrac{l}{m}\Big)\tfrac{\partial}{\partial y}\Big]^i f\Big(\tfrac{k}{n}, \tfrac{l}{m}\Big) \frac{\Big(nx\Big)^k}{k!} \frac{\Big(my\Big)^l}{l!} = 1,$$
$$f^{(0,0)} = f \equiv 1,$$
$$f^{(i,j)}\Big(\frac{k}{n}, \frac{l}{m}\Big) = 0, \qquad i \geq 1, j \geq 0.$$

It is obvious that

$$S_{n,m,r}(f;x,y) = S_{n,m,r}(t;x,y) = \frac{k}{n} + x - \frac{k}{n} = x,$$
$$f^{(0,0)} = f\Big(\frac{k}{n}, \frac{l}{m}\Big) \equiv \frac{k}{n},$$
$$f^{(1,0)}\Big(\frac{k}{n}, \frac{l}{m}\Big) = 1,$$
$$f^{(i,j)}\Big(\frac{k}{n}, \frac{l}{m}\Big) = 0, \qquad i \geq 2, j \geq 0.$$

Further, we have

$$S_{n,m,r}(f;x,y) = S_{n,m,r}(\tau;x,y) = \frac{l}{m} + y - \frac{l}{m} = y,$$
$$f^{(0,0)} = f\Big(\frac{k}{n}, \frac{l}{m}\Big) \equiv \frac{l}{m},$$
$$f^{(1,0)}\Big(\frac{k}{n}, \frac{l}{m}\Big) = 1,$$
$$f^{(i,j)}\Big(\frac{k}{n}, \frac{l}{m}\Big) = 0, \qquad i \geq 2, j \geq 0.$$

This complete the proof Lemma 2.

Remark 2. *In one-dimensional case a Korovkin's type theorems in the space $C_\rho\Big(R\Big)$ was proved in [5], where shown that in general the convergence in norm of the space $C_\rho\Big(R\Big)$ not holds.*

3. Main result

It is known that usual first modulus of continuity $\omega\Big(f;\delta\Big)$ does not tends to zero as $\delta\to 0$ on the infinite interval. Now we give weighted modulus of continuity $\Omega\Big(f;\delta\Big)$ which tens to zero as $\delta\to 0$ on the infinite interval.

Let $R_+^2 =: \Big\{(x,y)\in R^2; x\geq 0, y\geq 0\Big\}$ and$\rho\Big(x,y\Big) = 1+x^m+y^m$is weight function. Assume $B_\rho\Big(R_+^2\Big) := \Big\{f : \Big|f(x,y)\Big| \leq M_f\rho(x,y)\Big\}$, $M_f>0$and by $C^{(r)}(R_+^2)$ we denote the class of r-times continuously differentiable functions on R_+^2. We denote $C_\rho\Big(R_+^2\Big) := \{f : f\in B_\rho \bigcap C(R_+^2)\}$

$$C_\rho^K\Big(R_+^2\Big) := \Big\{f : f\in C_\rho\Big(R_+^2\Big) \lim_{x+y\to+\infty}\frac{f(x,y)}{\rho(x,y)} = K_f < \infty\Big\},$$

$$C_\rho^0\Big(R_+^2\Big) := \Big\{f : f\in C_\rho^K\Big(R_+^2\Big) \lim_{x+y\to+\infty}\frac{f(x,y)}{\rho(x,y)} = 0\Big\}.$$

It is well known that the norm in the spacesB_ρ is defined as

$$\Big\|f\Big\|_\rho = \sup_{(x,y)\in R_+^2}\frac{\Big|f(x,y)\Big|}{\rho(x,y)}.$$

Obviously, $C_\rho^K\Big(R_+^2\Big)\subset C_\rho\Big(R_+^2\Big)\subset B_\rho\Big(R_+^2\Big)$.

Remark 3. *Note that for functions of one variable the property of weighted modulus of continuity were studied in the work A.D. Gadjiev [2, 4], A. Aral [1] and etc.*

Now we consider a modulus of continuity of two variables functions on R_+^2 defined as:

$$\Omega\Big(f;\delta_1;\delta_2\Big) = \Omega\Big(\delta_1;\delta_2\Big) = \sup_{\substack{(x,y)\in R_+^2 \\ \big|h_1\big|\leq\delta_1, \big|h_2\big|\leq\delta_2}}\frac{\Big|f\Big(x+h_1,y+h_2\Big) - f\Big(x,y\Big)\Big|}{\rho\Big(x,y\Big)\rho\Big(\Big|h_1\Big|,\Big|h_2\Big|\Big)}. \tag{3}$$

Theorem 1. *Let $f\in C_\rho^K\Big(R_+^2\Big)$. Then for* (3) *the equality*

$$\lim_{\substack{\delta_1\to 0\\ \delta_2\to 0}}\Omega(\delta_1,\delta_2) = 0, \tag{4}$$

holds. Also the following properties is valid:
1) $\Omega\Big(0;0\Big) = 0$;
2) $\Omega(\delta_1;\delta_2)$ *is non-increasing with respect to* δ_1 *and* δ_2*;*
3) $\Omega\Big(\delta_{11}+\delta_{12},\delta_{21}+\delta_{22}\Big)\leq\Omega\Big(\delta_{11},\delta_{21}\Big)+\Omega\Big(\delta_{12},\delta_{22}\Big)$ *is semiadditive;*
4) $\Omega(\delta_1,\delta_2)$ *is continuous.*

Proof. We have

$$\Omega(\delta_1,\delta_2) \le \sup_{\substack{0<x\le x_0,\,0<y\le y_0\\ \left|h_1\right|\le\delta_1,\,\left|h_2\right|\le\delta_2}} \frac{\left|f\left(x+h_1,y+h_2\right)-f\left(x,y\right)\right|}{\rho\left(x,y\right)\rho\left(\left|h_1\right|,\left|h_2\right|\right)}$$

$$+ \sup_{\substack{0<x\le x_0,\,y>y_0\\ \left|h_1\right|\le\delta_1,\,\left|h_2\right|\le\delta_2}} \frac{\left|f\left(x+h_1,y+h_2\right)-f\left(x,y\right)\right|}{\rho\left(x,y\right)\rho\left(\left|h_1\right|,\left|h_2\right|\right)}+$$

$$+ \sup_{\substack{x>x_0,\,0<y\le y_0\\ \left|h_1\right|\le\delta_1,\,\left|h_2\right|\le\delta_2}} \frac{\left|f\left(x+h_1,y+h_2\right)-f\left(x,y\right)\right|}{\rho\left(x,y\right)\rho\left(\left|h_1\right|,\left|h_2\right|\right)}$$

$$+ \sup_{\substack{x>x_0,\,y>y_0\\ \left|h_1\right|\le\delta_1,\,\left|h_2\right|\le\delta_2}} \frac{\left|f\left(x+h_1,y+h_2\right)-f\left(x,y\right)\right|}{\rho\left(x,y\right)\rho\left(\left|h_1\right|,\left|h_2\right|\right)} = I_1+I_2+I_3+I_4.$$

It is obvious that

$$I_1 \le \sup_{\substack{0<x\le x_0,\,0<y\le y_0\\ \left|h_1\right|\le\delta_1,\,\left|h_2\right|\le\delta_2}} \left|f\left(x+h_1,y+h_2\right)-f\left(x,y\right)\right| \le \omega\left(\delta_1,\delta_2\right),$$

where

$$\omega\left(\delta_1,\delta_2\right) = \sup_{\substack{\left(x,y\right)\in R_+^2\\ \left|h_1\right|\le\delta_1,\,\left|h_2\right|\le\delta_2}} \left|f\left(x+h_1,y+h_2\right)-f\left(x,y\right)\right|,$$

is usual modulus of continuity of function f. We get

$$\frac{\left|f(x+h_1,y+h_2)-f(x,y)\right|}{\left(1+\left|h_1\right|^m+\left|h_2\right|^m\right)\left(1+x^m+y^m\right)} = \Big|\frac{f(x+h_1,y+h_2)}{\left(1+\left|h_1\right|^m+\left|h_2\right|^m\right)\left(1+x^m+y^m\right)}$$

$$-\frac{f(x,y)}{\left(1+\left|h_1\right|^m+\left|h_2\right|^m\right)\left(1+x^m+y^m\right)}\Big| = \Big|\frac{f(x+h_1,y+h_2)}{\left(1+\left|h_1\right|^m+\left|h_2\right|^m\right)\left(1+x^m+y^m\right)}$$

$$-\frac{K_f}{1+\left|h_1\right|^m+\left|h_2\right|^m}+\frac{K_f}{1+\left|h_1\right|^m+\left|h_2\right|^m}\frac{f(x,y)}{\left(1+\left|h_1\right|^m+\left|h_2\right|^m\right)\left(1+x^m+y^m\right)}\Big|$$

$$\leq \Big|\frac{f(x+h_1,y+h_2)}{\left(1+\left|h_1\right|^m+\left|h_2\right|^m\right)\left(1+x^m+y^m\right)}-\frac{K_f}{1+\left|h_1\right|^m+\left|h_2\right|^m}\Big|$$

$$+\Big|\frac{f(x,y)}{\left(1+\left|h_1\right|^m+\left|h_2\right|^m\right)\left(1+x^m+y^m\right)}-\frac{K_f}{1+\left|h_1\right|^m+\left|h_2\right|^m}\Big|$$

$$=\frac{1}{1+\left|h_1\right|^m+\left|h_2\right|^m}\left(\Big|\frac{f(x+h_1,y+h_2)}{1+x^m+y^m}-K_f\Big|+\Big|\frac{f(x,y)}{1+x^m+y^m}-K_f\Big|\right)$$

$$=\frac{1}{1+\left|h_1\right|^m+\left|h_2\right|^m}\left(\Big|\frac{f(x+h_1,y+h_2)}{1+x^m+y^m}-\frac{f(x+h_1,y+h_2)}{1+(x+\left|h_1\right|^m)+\left(y+\left|h_2\right|\right)^m}\right.$$

$$\left.+\frac{f(x+h_1,y+h_2)}{1+(x+\left|h_1\right|^m)+\left(y+\left|h_2\right|\right)^m}-K_f\Big|+\Big|\frac{f(x,y)}{1+x^m+y^m}-K_f\Big|\right.$$

$$\leq\frac{1}{1+\left|h_1\right|^m+\left|h_2\right|^m}\left(\Big|\frac{f(x+h_1,y+h_2)}{1+x^m+y^m}-\frac{f(x+h_1,y+h_2)}{1+(x+\left|h_1\right|^m)\left(y+\left|h_2\right|\right)^m}\Big|\right.$$

$$\left.+\Big|\frac{f(x+h_1,y+h_2)}{1+(x+\left|h_1\right|^m)+\left(y+\left|h_2\right|\right)^m}-K_f\Big|\right)+\Big|\frac{f(x,y)}{1+x^m+y^m}-K_f\Big|\Big)$$

$$=\frac{1}{1+\left|h_1\right|^m+\left|h_2\right|^m}\left(\frac{\left|(x+h_1)^m+(y+h_2)^m-x^m-y^m\right|}{\left(1+x^m+y^m\right)\left(1+(x+\left|h_1\right|^m)+\left(y+\left|h_2\right|\right)^m\right)}\left|f(x+h)\right|\right.$$

$$\left.+\Big|\frac{f(x+h_1,y+h_2)}{1+(x+\left|h_1\right|^m)+\left(y+\left|h_2\right|\right)^m}-K_f\Big|+\Big|\frac{f(x,y)}{1+x^m+y^m}-K_f\Big|\right)$$

$$\leq\frac{\left|(x+h_1)^m+(y+h_2)^m-x^m-y^m\right|}{1+x^m+y^m}\frac{\left|f(x+h_1,y+h_2)\right|}{1+(x+\left|h_1\right|^m)+\left(y+\left|h_2\right|\right)^m}$$

$$+\Big|\frac{f(x,y)}{1+(x+\left|h_1\right|^m)+\left(y+\left|h_2\right|\right)^m}-K_f\Big|+\Big|\frac{f(x,y)}{1+x^m+y^m}-K_f\Big|. \tag{5}$$

By the mean theorem, we get

$$\frac{\left|f(x+h_1,y+h_2)\right|}{1+(x+\left|h_1\right|)^m+(y+\left|h_2\right|)^m}\ \frac{1}{1+x^m+y^m}\left|(x+h_1)^m+(y+h_2)^m-x^m-y^m\right|\leq$$

$$\leq \frac{\Big|f(x+h_1,y+h_2)\Big|}{1+(x+\Big|h_1\Big|)^m+(y+\Big|h_2\Big|)^m}\ \frac{1}{1+x^m+y^m}\Big|\xi^{m-1}mh_1+\eta^{m-1}mh_2\Big|\leq$$

$$\leq \frac{m\Big(x^{m-1}+(x+h_1)^{m-1}+y^{m-1}+\Big(y+h_2\Big)^{m-1}\Big)}{1+x^m+y^m}$$

$$\Big(\Big|\frac{f(x+h_1,y+h_2)}{1+(x+\Big|h_1\Big|^m)+\Big(y+\Big|h_2\Big|\Big)^m}-K_f\Big|+\Big|K_f\Big|\Big).$$

(5) implies that

$$\frac{\Big|f(x+h_1,y+h_2)-f(x,y)\Big|}{\Big(1+(x+\Big|h_1\Big|^m)+\Big(y+\Big|h_2\Big|\Big)^m)(1+x^m+y^m\Big)}$$

$$\leq\Big(1+\frac{m\Big(x^{m-1}+(x+h)^{m-1}+y^{m-1}+\Big(y+h_2\Big)^{m-1}\Big)}{1+x^m+y^m}\Big)$$

$$\times\Big(\Big|\frac{f(x+h_1,y+h_2)}{1+(x+\Big|h_1\Big|^m)+\Big(y+\Big|h_2\Big|\Big)^m}-K_f\Big|\Big)+\Big|\frac{f(x,y)}{1+x^m+y^m}-K_f\Big|$$

$$+\frac{m\Big(x^{m-1}+(x+h)^{m-1}+y^{m-1}+\Big(y+h_2\Big)^{m-1}\Big)}{1+x^m+y^m}\Big|K_f\Big|$$

$$\leq\Big(1+\frac{m\Big(x^{m-1}+y^{m-1}+2^{m-2}\Big(x^{m-1}+y^{m-1}+\Big|h_1\Big|^{m-1}+\Big|h_2\Big|^{m-1}\Big)\Big)}{1+x^m+y^m}\Big)$$

$$\times\Big|\frac{f(x+h_1,y+h_2)}{1+(x+\Big|h_1\Big|^m)+\Big(y+\Big|h_2\Big|\Big)^m}-K_f\Big|$$

$$+\Big|\frac{f(x,y)}{1+x^m+y^m}-K_f\Big|+\frac{m(x^{m-1}+y^{m-1}+(x+h_1)^{m-1}+\Big(y+h_2\Big)^{m-1}}{x^m+y^m}\Big|K_f\Big|.$$

$$\frac{\Big|f(x+h_1,y+h_2)-f(x,y)\Big|}{\Big(1+(x+\Big|h_1\Big|^m)+\Big(y+\Big|h_2\Big|\Big)^m\Big)\Big(1+x^m+y^m\Big)}\leq$$

$$\leq m2^m\Big|\frac{f(x+h_1,y+h_2)}{1+(x+\Big|h_1\Big|^m)+\Big(y+\Big|h_2\Big|\Big)^m}-K_f\Big|$$

$$+\Big|\frac{f(x+h_1,y+h_2)}{1+x^m+y^m}-K_f\Big|+(1+2^m)\Big(\frac{1}{x}+\frac{1}{y}\Big)\Big|K_f\Big|.$$

We estimate I_2. We have

$$\frac{\left|f(x+h_1,y+h_2)-f(x,y)\right|}{(1+\left|h_1\right|^m+\left|h_2\right|^m)(1+x^m+y^m)}\le\frac{\left|f(x+h_1,y+h_2)-f(x,y)\right|}{(1+\left|h_1\right|^m+\left|h_2\right|^m)(1+y^m)}.$$

Analogously to (5) for $0<x\le x_0$ and large $y\in R_+$, we have

$$\frac{\left|f(x+h_1,y+h_2)-f(x,y)\right|}{(1+\left|h_1\right|^m+\left|h_2\right|^m)(1+y^m)}\le m2^m\Big|\frac{f(x+h_1,y+h_2)}{1+(y+h_2)^m}-$$

$$-K_f\Big|+\Big|\frac{f(x,y)}{1+y^m}-K_f\Big|+\frac{m2^m}{y}\Big|K_f\Big|$$

Analogously to (5) for $0<y\le y_0$ and large $x\in R_+$, we have

$$\frac{\left|f(x+h_1,y+h_2)-f(x,y)\right|}{(1+\left|h_1\right|^m+\left|h_2\right|^m)(1+x^m+y^m)}\le\frac{\left|f(x+h_1,y+h_2)-f(x,y)\right|}{(1+\left|h_1\right|^m+\left|h_2\right|^m)(1+x^m)}$$

$$\le m2^m\Big|\frac{f(x+h_1,y+h_2)}{1+(x+h_1)^m}-K_f\Big|+\Big|\frac{f(x,y)}{1+x^m}-K_f\Big|+\frac{m2^m}{x}\Big|K_f\Big|.$$

Finally we estimate I_4 for $x,y\to\infty$. Analogously to (5), we get

$$\frac{\left|f(x+h_1,y+h_2)-f(x,y)\right|}{(1+\left|h_1\right|^m+\left|h_2\right|^m)(1+x^m+y^m)}\le$$

$$\le m2^m\Big|\frac{f(x+h_1,y+h_2)}{1+(x+h_1)^m+(y+h_2)^m}-K_f\Big|+$$

$$+\Big|\frac{f(x,y)}{1+x^m+y^m}-K_f\Big|+m2^m\Big(\frac{1}{x}+\frac{1}{y}\Big)\Big|K_f\Big|$$

Therefore, by the definition of $C_p(R_+)$for given$\varepsilon>0$, we can choose $x_0=x_0(\varepsilon)$ and $y_0=y_0(\varepsilon)$ such that the inequalities

$$I_2<\frac{\varepsilon}{3};\quad I_3<\frac{\varepsilon}{3};\quad I_4<\frac{\varepsilon}{3}.$$

Thus

$$\Omega(\delta_1,\delta_2)\le\omega(\delta_1,\delta_2)+\varepsilon.$$

Since $\overline{\lim\limits_{\substack{\delta_1\to+0\\ \delta_2\to+0}}}\,\Omega\Big(\delta_1,\delta_2\Big)<\varepsilon$ for any $\varepsilon>0$, we get $\overline{\lim\limits_{\substack{\delta_1\to+0\\ \delta_2\to+0}}}\,\Omega\Big(\delta_1,\delta_2\Big)=0$.

Note that the property 2) is obvious. The properties 3) and 4) is proved analogously to the one-dimensional case.

This completes the proof.

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