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Generating Function of the Number of Jumps at which Complex Process of Semi-Markov Walk Achieves First the Level "a"$(a > 0)$

T.I. Nasirova*, E.M. Neymanov, U.Y. Kerimova

Abstract. Using the sequence of independent random variables, we construct difference process of semi-markov walk. The generating function of the number of jumps under which complex process of semi-markov walk achieves first the level "a" $(a > 0)$, is found.

Key Words and Phrases: random variable, process of semi-markov walk, generating function.

2010 Mathematics Subject Classifications: 60A10; 60J25; 60D10

1. Introduction

There are a few papers devoted to studying generating function of the number of jumps under which complex process of semi-markov walk achieves first the level "a" $(a > 0)$.

In the paper [1,p. 61-63], asymptotic behavior of random walks in a random medium with delaying barrier was studied. Random walk in a band was studied in [2, p. 160-165]. In [3, p. 26-51], asymptotic expansion of distributions determined on Markov chains, was found. Different semi-makrov chains with delaying barrier and functionals of these processes were studied in the paper [4, p. 61-63]. In [5,p. 77-84], Laplace transform of distribution of the lower boundary functional of the process of semi-markov walk with delaying barrier in zero, was found. The Laplace transform of ergodic distribution of the process of semi-markov walk with negative drift, nonnegative jumps and delaying barrier in zero was found in [6,p. 49-60].

In the present paper we find a generating function of the number of jumps under which the complex process of semi-markov walk achieves first the level "a" $(a > 0)$.

As far as we know, a generating function of the number of jumps at which it achieves first the level "a" $(a > 0)$ was not found for complex process of semi-markov walk.

*Corresponding author.

2. Mathematical statement of the problem.

On probability space $(\Omega, F, P(\cdot))$ we are given the sequence of independent identically distributed positive random variables $\xi_k^+, \eta_k^+, \xi_k^-, \eta_k^-$, $k = \overline{1, \infty}$.

Introduce the following denotation $\nu\pm(t) = \min\left\{k : \sum\limits_{i=1}^{k+1} \xi_i^\pm > t\right\}$ is the number of positive jumps of the process $\mathrm{X}^\pm(t)$ for time t

$$\mathrm{X}^\pm(t) = \sum_{i=1}^{\nu^\pm(t)} \eta_i^\pm$$

$$\mathrm{X}(t) = \mathrm{X}^+(t) - \mathrm{X}^-(t)$$

The process $\mathrm{X}(t) = \mathrm{X}^+(t) - \mathrm{X}^-(t)$ is called a complex process of semi-markov walk.

The goal of the paper is to find explicit form of the generating function of the number of jumps under which the process $X(t)$ achieves first the level "a".

We denote it by ν_1^a.

Let $X(0) = z > 0$.

3. Setting-up integral equation for generating function of the number of jumps of the process $X(t)$ under which it achieves first the level $''a''(a > 0)$.

Denote by ν_1^a the number of jumps of the process $X(t)$ under which it achieves first the level "a"$(a > 0)$.

Denote

$$\Psi(u|z) = \sum_{k=1}^{\infty} u^k P\left\{\nu_1^a = k | X(0) = z\right\}, |u| \leq 1$$

Theorem 1. $\Psi(u|z)$ *satisifies the following integral equation*

$$\Psi(u|z) = uP\left\{\eta_1^+ > a - z\right\} + u\int_{y=z}^{a} \Psi(u|y)d_yP\left\{\eta_1^+ < y - z\right\}P\left\{\xi_1^+ < \xi_1^-\right\} +$$

$$+u\int_{y=z}^{a} \Psi(u|y)\int_{x=y}^{\infty} dy\sum_{m=1}^{\infty} P\left\{\eta_1^- + ... + \eta_m^- < x - y\right\} \times$$

$$\times\int_{t=0}^{\infty} P\left\{\nu^-(t) = m\right\} d_tP\left\{\xi_1^+ - \xi_1^- < t\right\} d_yP\left\{\eta_1^+ < y\right\} +$$

$$+u\int\limits_{y=-\infty}^{z}\Psi(u|y)\int\limits_{x=z}^{\infty}dy\sum_{m=1}^{\infty}P\left\{\eta_1^-+...+\eta_m^-<x-y\right\}\times$$

$$\times\int\limits_{t=0}^{\infty}P\left\{\nu^-(t)=m\right\}d_tP\left\{\xi_1^+-\xi_1^-<t\right\}d_yP\left\{\eta_1^+<y\right\}. \qquad (1)$$

Proof. Let $k\geq 2$. Then by the total probability formula we have

$$\begin{aligned}
&P\left\{\nu_1^a=k|X(0)=z\right\}=P\left\{\nu_1^a=k;(\xi_1^+<\xi_1^-)\bigcup(\xi_1^-<\xi_1^+)|X(0)=z\right\}=\\
&=P\left\{\nu_1^a=k;(\xi_1^+<\xi_1^-)|X(0)=z\right\}+P\left\{\nu_1^a=k;(\xi_1^-<\xi_1^+)|X(0)=z\right\}=\\
&=\int\limits_{y=z}^{a}P\left\{\xi_1^+<\xi_1^-;z+\eta_1^+<a;z+\eta_1^-\in dy\right\}P\left\{\nu_1^a=k-1|X(0)=y\right\}+\\
&+\int\limits_{y=z}^{a}P\left\{\xi_1^-<\xi_1^+;z-\eta_1^--\eta_2^--...-\eta_{\nu^-(\xi_1^+-\xi_1^-)}+\eta_1^+\in dy\right\}P\left\{\nu_1^a=k-1|X(0)=y\right\}
\end{aligned}.$$

So, by $x-y>0$ and $x-z>0$ we get

$$P\left\{\nu_1^a=k|X(0)=z\right\}=\int\limits_{y=z}^{a}P\left\{\nu_1^a=k-1|X(0)=y\right\}d_yP\left\{\eta_1^+<y-z\right\}P\left\{\xi_1^+<\xi_1^-\right\}-$$

$$-\int\limits_{y=z}^{a}P\left\{\nu_1^a=k-1|X(0)=y\right\}\int\limits_{x=\max(y,z)}^{\infty}d_y\sum_{m=1}^{\infty}P\left\{\eta_1^-+...+\eta_m^-<x-y\right\}$$

$$\int\limits_{t=0}^{\infty}P\left\{\nu^-(t)=m\right\}d_tP\left\{\xi_1^+-\xi_1^-<t\right\}d_xP\left\{\eta_1^+<x-z\right\}.$$

Then we have

$$P\left\{\nu_1^a=k|X(0)=z\right\}=\int\limits_{y=z}^{a}P\left\{\nu_1^a=k-1|X(0)=y\right\}d_yP\left\{\eta_1^+<y-z\right\}P\left\{\xi_1^--\xi_1^+<t\right\}-$$

$$-\int\limits_{y=z}^{a}P\left\{\nu_1^a=k-1|X(0)=y\right\}\int\limits_{x=y}^{\infty}d_yP\sum_{m=1}^{\infty}P\left\{\eta_1^-+...+\eta_m^-<x-y\right\}$$

$$\int\limits_{t=0}^{\infty}P\left\{\nu^-(t)=m\right\}d_tP\left\{\xi_1^+-\xi_1^-<t\right\}d_xP\left\{\eta_1^+<x-z\right\}+$$

$$-\int_{y=-\infty}^{z} P\left\{\nu_1^a = k-1|X(0)=y\right\} \int_{x=z}^{\infty} d_y P \sum_{m=1}^{\infty} P\left\{\eta_1^- + ... + \eta_m^- < x-y\right\}$$

$$\int_{t=0}^{\infty} P\left\{\nu^-(t)=m\right\} d_t P\left\{\xi_1^+ - \xi_1^- < t\right\} d_y P\left\{\eta_1^+ < y-z\right\}$$

We multiply the both hand sides of (1) by u^k and sum over $k \geq 2$.

$$\sum_{k=2}^{\infty} u^k P\left\{\nu_1^a = k|X(0)=z\right\} =$$

$$= \int_{y=z}^{a} \sum_{k=2}^{\infty} u^k P\left\{\nu_1^a = k-1|X(0)=y\right\} d_y P\left\{\eta_1^+ < y-z\right\} P\left\{\xi_1^+ < \xi_1^-\right\} +$$

$$-\int_{y=z}^{a} \sum_{k=2}^{\infty} u^k P\left\{\nu_1^a = k-1|X(0)=y\right\} \times$$

$$\times \int_{x=y}^{\infty} d_y \sum_{m=1}^{\infty} P\left\{\eta_1^- + ... + \eta_m^- < x-y\right\}$$

$$\int_{t=0}^{\infty} P\left\{\nu^-(t)=m\right\} d_t P\left\{\xi_1^+ - \xi_1^- < t\right\} d_x P\left\{\eta_1^+ < x-z\right\} -$$

$$-\int_{y=-\infty}^{z} \sum_{k=2}^{\infty} u^k P\left\{\nu_1^a = k-1|X(0)=y\right\} \int_{x=z}^{\infty} d_y \sum_{m=1}^{\infty} P\left\{\eta_1^- + ... + \eta_m^- < x-y\right\} \times$$

$$\times \int_{t=0}^{\infty} P\left\{\nu^-(t)=m\right\} d_t P\left\{\xi_1^+ - \xi_1^- < t\right\} d_y P\left\{\eta_1^+ < y-z\right\}. \qquad (2)$$

Obviously,

$$P\{z + \eta_1^+ > a\} = P\{\nu_1^a = 1|X(0)=z\}. \qquad (3)$$

Adding (3) to both hand sides of (2), we complete the proof of the theorem.

We will solve the equation with respect to $\Psi(u|z)$ if the random variables ξ_k^+, η_k^+, ξ_k^-, η_k^- have exponential distribution with the parameters $\lambda_+ > 0$, $\lambda_- > 0$, $\mu_+ > 0$, $\mu_- > 0$, respectively.

$$P\{\xi_1^\pm < t\} = \begin{cases} 0, t < 0 \\ 1 - e^{-\lambda_\pm t} \end{cases}$$

$$t > 0, \lambda_+ > 0, \lambda_- > 0 P\{\eta_1^\pm < x\} = \begin{cases} 0, x < 0 \\ 1 - e^{-\mu_\pm t} \end{cases} \quad x > 0, \mu_+ > 0, \mu_- > 0. \tag{4}$$

It is easy to find that under supposition (4),

$$P\left\{\xi_1^+ < \xi_1^-\right\} = \frac{\lambda_+}{\lambda_+ + \lambda_-},$$

$$P\left\{\xi_1^- < \xi_1^+\right\} = \frac{\lambda_-}{\lambda_+ + \lambda_-},$$

$$d_t P\left\{\xi_1^+ - \xi_1^- < t\right\} = \frac{\lambda_+\lambda_-}{\lambda_+ + \lambda_-} e^{-\lambda_+ t} dt.$$

From references it is known that

$$P\left\{\nu^\pm(t) = m\right\} = \frac{(\lambda_\pm t)^m}{m!} e^{-\lambda_\pm t}$$

$$d_y P\left\{\eta_1^- + \eta_2^- + ... + \eta_m^- < y\right\} = \mu_- \frac{(\mu_- y)^{m-1}}{(m-1)!} e^{-\mu_- y} dy.$$

We substitute these formuls in equation (1)

$$\Psi(u|z) = ue^{-\mu_+ a} e^{\mu_+ z} + \frac{\lambda_+\mu_+}{\lambda_+ + \lambda_-} ue^{\mu_+ z} \int\limits_{y=z}^{a} e^{-\mu_+ y} \Psi(u|y) dy +$$

$$+ \frac{\lambda_+\lambda_-^2\mu_+\mu_-}{(\lambda_+ + \lambda_-)^3} ue^{\mu_+ z} \int\limits_{y=-\infty}^{z} e^{\mu_- y} \Psi(u|y) \int\limits_{x=z}^{\infty} e^{-(\mu_+ + \mu_-)x} e^{\frac{\lambda_-\mu_-(x-y)}{\lambda_+ + \lambda_-}} dx dy +$$

$$+ \frac{\lambda_+\lambda_-^2\mu_+\mu_-}{(\lambda_+ + \lambda_-)^3} ue^{\mu_+ z} \int_{y=z}^{\infty} e^{\mu_- y} \Psi(u|y) \int\limits_{x=y}^{\infty} e^{-(\mu_+ + \mu_-)x} e^{\frac{\lambda_-\mu_-(x-y)}{\lambda_+ + \lambda_-}} dx dy \tag{5}$$

Having multiplied the both hand sides by $e^{-\mu_, z}$ and differentiated with respect to z, we get

$$\Psi'(u|z) - \mu_+ \Psi(u|z) = -\frac{\lambda_+\mu_+}{\lambda_+ + \lambda_-} u\Psi(u|z) -$$

$$- \frac{\lambda_+\lambda_-^2\mu_+\mu_-}{(\lambda_+ + \lambda_-)^3} ue^{(\frac{\lambda_-\mu_-}{\lambda_+ + \lambda_-} - \mu_-)z} \int\limits_{y=-\infty}^{z} e^{\mu_- y} \Psi(u|y) e^{-\frac{\lambda_-\mu_-}{\lambda_+ + \lambda_-} y} dy. \tag{6}$$

We multiply the both hand sides by $e^{(\mu_- - \frac{\lambda_-\mu_-}{\lambda_++\lambda_-})z}$

$$\left[\Psi'(u|z) - \mu_+\Psi(u|z)\right] e^{(\mu_- - \frac{\lambda_-\mu_-}{\lambda_++\lambda_-})z} = -\frac{\lambda_+\mu_+}{\lambda_+ + \lambda_-} u\Psi(u|z)e^{(\mu_- - \frac{\lambda_-\mu_-}{\lambda_++\lambda_-})z} -$$

$$-\frac{\lambda_+\lambda_-^2\mu_+\mu_-}{(\lambda_+ + \lambda_-)^3} u \int\limits_{y=-\infty}^{z} e^{\mu_- y}\Psi(u|y)e^{-\frac{\lambda_-\mu_-}{\lambda_++\lambda_-}y} dy.$$

Differentiate both hand sides with respect to z.

$$\left[(\Psi'(u|z) - \mu_+\Psi(u|z))(\mu_- - \frac{\lambda_-\mu_-}{\lambda_+ + \lambda_-}) + \Psi''(u|z) - \mu_+\Psi'(u|z)\right] e^{(\mu_- - \frac{\lambda_-\mu_-}{\lambda_++\lambda_-})z} =$$

$$-\frac{\lambda_+\mu_+}{\lambda_+ + \lambda_-} u e^{(\mu_- - \frac{\lambda_-\mu_-}{\lambda_++\lambda_-})z} [\Psi'(u|z) + (\mu_- - \frac{\lambda_-\mu_-}{\lambda_+ + \lambda_-})\Psi(u|z)] -$$

$$-\frac{\lambda_+\lambda_-^2\mu_+\mu_-}{(\lambda_+ + \lambda_-)^3} u e^{\mu_- z}\Psi(u|z)e^{-\frac{\lambda_-\mu_-}{\lambda_++\lambda_-}z}.$$

Multiply both hand sides by $e^{-(\mu_- - \frac{\lambda_-\mu_-}{\lambda_++\lambda_-})z}$

$$(\mu_- - \frac{\lambda_-\mu_-}{\lambda_+ + \lambda_-})[\Psi'(u|z) - \mu_+\Psi(u|z)] + \Psi''(u|z) - \mu_+\Psi'(u|z)$$

$$= -\frac{\lambda_+\mu_+}{\lambda_+ + \lambda_-} u[\Psi'(u|z) + (\mu_- - \frac{\lambda_-\mu_-}{\lambda_+ + \lambda_-})\Psi(u|z)] - \frac{\lambda_+\lambda_-^2\mu_+\mu_-}{(\lambda_+ + \lambda_-)^3} u\Psi(u|z).$$

We get a second order homogeneous differential equation

$$\Psi''(u|z) + [-\mu_+ + \frac{\lambda_+\mu_-}{\lambda_+ + \lambda_-} + \frac{\lambda_+\mu_+}{\lambda_+ + \lambda_-} u]\Psi'(u|z) + [-\frac{\lambda_+\mu_+\mu_-}{\lambda_+ + \lambda_-} + \frac{\lambda_+^2\mu_+\mu_-}{(\lambda_+ + \lambda_-)^2} u +$$

$$+\frac{\lambda_+\lambda_-^2\mu_+\mu_-}{(\lambda_+ + \lambda_-)^3} u]\Psi(u|z) = 0. \tag{7}$$

Let us solve this equation.
Characteristic equation and the roots

$$K^2(u) + [-\mu_+ + \frac{\lambda_+\mu_-}{\lambda_+ + \lambda_-} + \frac{\lambda_+\mu_+}{\lambda_+ + \lambda_-} u] \times$$

$$\times K(u) + [-\frac{\lambda_+\mu_+\mu_-}{\lambda_+ + \lambda_-} + \frac{\lambda_+^2\mu_+\mu_-}{(\lambda_+ + \lambda_-)^2} u + \frac{\lambda_+\lambda_-^2\mu_+\mu_-}{(\lambda_+ + \lambda_-)^3} u] = 0 \tag{8}$$

$$K_{1,2}(1)=\frac{\frac{\lambda_+\mu_-+\lambda_-\mu_+}{\lambda_++\lambda_-}\pm\frac{1}{\lambda_++\lambda_-}\sqrt{\lambda_+^2\mu_-^2+\lambda_-^2\mu_+^2+\frac{2\lambda_+\lambda_-\mu_+\mu_-(\lambda_+-\lambda_-)}{\lambda_++\lambda_-}}}{2}$$

if $u=1$ we get,

$$K_{1,2}(1)=\frac{\lambda_+\mu_-+\lambda_-\mu_+\pm\sqrt{\lambda_+^2\mu_-^2+\lambda_-^2\mu_+^2+\frac{2\lambda_+\lambda_-\mu_+\mu_-(\lambda_+-\lambda_-)}{\lambda_++\lambda_-}}}{2(\lambda_++\lambda_-)}$$

We get the solution of the differential equation

$$\Psi(u|z)=C_1(u)e^{k_1(u)z}+C_2(u)e^{k_2(u)z} \tag{9}$$

If in equations (5) and (6) we substitute $z=0$, we get the system of equations

$$\begin{cases} C_1(u)\Big[1-\frac{\lambda_+\mu_+u}{(\lambda_++\lambda_-)(k_1(u)-\mu_+)}\left(e^{(k_1-\mu_+)a}-1\right)- \\ -\frac{\lambda_-^2\mu_-u}{(\lambda_++\lambda_-)(k_1(u)(\lambda_++\lambda_-)+\lambda_+\mu_-)}+\frac{\lambda_+\lambda_-^2\mu_+\mu_-u}{(\lambda_++\lambda_-)^2(\mu_+(\lambda_++\lambda_-)+\lambda_+\mu_-(k_1(u)-\mu_+))}+\Big] \\ +C_2(u)\Big[1-\frac{\lambda_+\mu_+u}{(\lambda_++\lambda_-)(k_2(u)-\mu_+)}\left(e^{(k_2-\mu_+)a}-1\right)- \\ -\frac{\lambda_-^2\mu_-u}{(\lambda_++\lambda_-)(k_2(u)(\lambda_++\lambda_-)+\lambda_+\mu_-)}+\frac{\lambda_+\lambda_-^2\mu_+\mu_-u}{(\lambda_++\lambda_-)^2(\mu_+(\lambda_++\lambda_-)+\lambda_+\mu_-(k_2(u)-\mu_+))}\Big]= \\ =ue^{-\mu_+a} \\ C_1(u)\Big[k_1(u)-\mu_++\frac{\lambda_+\mu_+u}{\lambda_++\lambda_-}-\frac{\lambda_+\lambda_-^2\mu_+\mu_-u}{(\lambda_++\lambda_-)^2(k_1(u)(\lambda_++\lambda_-)+\lambda_+\mu_-)}\Big]+ \\ +C_2(u)\Big[k_2(u)-\mu_++\frac{\lambda_+\mu_+u}{\lambda_++\lambda_-}-\frac{\lambda_+\lambda_-^2\mu_+\mu_-u}{(\lambda_++\lambda_-)^2(k_2(u)(\lambda_++\lambda_-)+\lambda_+\mu_-)}\Big]=0 \end{cases}$$

Simplify the second equation of system (10). For that we use the characteristic equation and get

$$k_1(u)-\mu_++\frac{\lambda_+\mu_+u}{\lambda_++\lambda_-}-\frac{\lambda_+\lambda_-^2\mu_+\mu_-u}{(\lambda_++\lambda_-)^2[k_1(u)(\lambda_++\lambda_-)+\lambda_+\mu_-]}=0$$

$$k_2(u)-\mu_++\frac{\lambda_+\mu_+u}{\lambda_++\lambda_-}-\frac{\lambda_+\lambda_-^2\mu_+\mu_-u}{(\lambda_++\lambda_-)^2[k_2(u)(\lambda_++\lambda_-)+\lambda_+\mu_-]}=0$$

$$C_1(u)\Bigg[k_1^2(u)+k_1(u)[-\mu_++\frac{\lambda_+\mu_-}{\lambda_++\lambda_-}+\frac{\lambda_+\mu_+u}{\lambda_++\lambda_-}]-$$

$$-\frac{\lambda_+\mu_+\mu_-}{\lambda_++\lambda_-}+\frac{\lambda_+^2\mu_+\mu_-u}{(\lambda_++\lambda_-)^2}+\frac{\lambda_+\lambda_-^2\mu_+\mu_-u}{(\lambda_++\lambda_-)^3}\Bigg]+$$

$$+C_2(u)\Bigg[k_2^2(u)+k_2(u)[-\mu_++\frac{\lambda_+\mu_-}{\lambda_++\lambda_-}+\frac{\lambda_+\mu_+u}{\lambda_++\lambda_-}]-$$

$$\frac{\lambda_+\mu_+\mu_-}{\lambda_++\lambda_-}+\frac{\lambda_+^2\mu_+\mu_-u}{(\lambda_++\lambda_-)^2}+\frac{\lambda_+\lambda_-^2\mu_+\mu_-u}{(\lambda_++\lambda_-)^3}\Bigg]=0.$$

We get

$$C_1(u)\cdot 0 + C_2(u)\cdot 0 = 0$$

Then we substitute $C_2(u) = 0$ in the first equation and get an expression for $C_1(u)$

$$C_1(u) = e^{-\mu_+ a} / \left[1 - \frac{\lambda_+\mu_+}{(\lambda_+ + \lambda_-)(k_1(u) - \mu_+)} (e^{(k_1-\mu_+)a} - 1) - \right.$$

$$\left. - \frac{\lambda_-^2\mu_-}{(\lambda_+ + \lambda_-)[k_1(u)(\lambda_+ + \lambda_-) + \lambda_+\mu_-]} + \frac{\lambda_+\lambda_-^2\mu_+\mu_-}{(\lambda_+ + \lambda_-)^2(\mu_+(\lambda_+ + \lambda_-) + \lambda_+\mu_-(k_1(u) - \mu_+))} \right].$$

We simplify it using the roots of the characteristic equation and get

$$C_1(u) =$$

$$= \frac{e^{-\mu_+ a}}{\frac{\lambda_+\mu_+}{(\lambda_+ + \lambda_-)(k_1(u) - \mu_+)} e^{(k_1-\mu_+)a} - \frac{\lambda_-^2\mu_-(\lambda_-\mu_+ + \lambda_+\mu_-)(k_1(u)+\mu_+)}{(\lambda_+ + \lambda_-)[k_1(u)(\lambda_+ + \lambda_-) + \lambda_+\mu_-](\mu_+(\lambda_+ + \lambda_-) + \lambda_+\mu_-)(k_1(u) - \mu_+)}}$$

If we substitute the values of $C_1(u)$ and $C_2(u)$ in equation (9), we get

$$\Psi(u|z) =$$

$$= \frac{e^{k_1(u)z}}{\frac{\lambda_+\mu_+}{(\lambda_+ + \lambda_-)(k_1(u) - \mu_+)} e^{k_1(u)a} - \frac{\lambda_-^2\mu_-(\lambda_-\mu_+ + \lambda_+\mu_-)(k_1(u)+\mu_+)}{(\lambda_+ + \lambda_-)[k_1(u)(\lambda_+ + \lambda_-) + \lambda_+\mu_-](\mu_+(\lambda_+ + \lambda_-) + \lambda_+\mu_-)(k_1(u) - \mu_+)} e^{\mu_+ a}}$$

or

$$\Psi(u|z) = \frac{(\lambda_+ + \lambda_-)(k_1(u) - \mu_+)e^{k_1(u)z}}{\lambda_+\mu_+ e^{k_1(u)a} - \frac{\lambda_-^2\mu_-(\lambda_-\mu_+ + \lambda_+\mu_-)(k_1(u)+\mu_+)}{[k_1(u)(\lambda_+ + \lambda_-) + \lambda_+\mu_-](\mu_+(\lambda_+ + \lambda_-) + \lambda_+\mu_-)} e^{\mu_+ a}}$$

4. Conclusion

Using the sequence of independent random variables, we constructed difference process of semi-markov process. We found generating function of the number of jumps under which complex process of semi-markov walk achieves first the level $''a''(a > 0)$.

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Tamilla Nasirova
Institute of Cybernetics of NAS of Azeribaijan, 9, B.Vahabzade str., Az1141, Baku, Azerbaijan

Elbrus Neymanov
Institute of Mathematics and Mechanics of NAS of Azerbaijan, Az1141, Baku, Azerbaijan
E-mail: eneymanov@inbox.ru

Ulviyya Kerimova
Baku State University, 23 Z.Khalilov str., Az1148, Baku, Azerbaijan
E-mail: ulviyyekerimova@yahoo.com

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