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Synthesis of laws for driving a car on a trajectory

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Abstract: This work is devoted to issues related to the synthesis of fuzzy control on the trajectory of a car, using computer and simulation tools to configure it. In many works, fuzzy logic is used to implement intuitive ideas about the behavior of an object and existing experience in a nonlinear fuzzy control law, the formal analysis of whose properties (such as the stability of a closed-loop system) is, as a rule, impossible to carry out. In this study, unlike such works, fuzzy logic is used to replace the original nonlinear model with several linear ones, and, similarly, obtaining a fuzzy controller TS) based on several linear controls. Moreover, such a closed system can be formally studied for global asymptotically stable, which makes such an approach promising from the point of view of application in practice. To synthesize the controller, the work also uses the tools of the classical LQR optimization method.

Keywords: Fuzzy controller, Fuzzy logic, Non-linear model, Management system, Automation system.

1.INTRODUCTION

In the last century, since the advent of the car, it immediately became an important part of human daily life. With the development of modern industry in the field of mechanical engineering, the number of car owners in all countries of the world has been characterized by a significant increase, and over time, the problem of traffic has become impossible to ignore in urban planning. In classic models, the “car-driver-road” system is a closed-loop system with feedback. The increasing volume of need to use the capabilities of modern cars, growing requirements for comfort and safety, make relevant a direction of research focused on how to exclude humans from this system. At all stages of automation, the task of studying the dynamics of vehicle motion is necessary, and the research must be based on knowledge of control theory and modeling, as well as the capabilities of modern computer and information technologies [1-7].

2. EXPERIMENTAL DETAIL

In this work, we will refer to the linear integral quadratic optimization (LQR optimization) method, which is popular in research practice, when synthesizing the control to ensure the movement along the trajectory. A description of this approach can be found, for example, in the monograph [1].

The LQR optimization problem is formulated for linear systems with mathematical models in the form of LTI systems (linear time-invariant)

$$\dot{x} = \boldsymbol{A}x + \boldsymbol{B}u, \qquad (1.1)$$

Where $x \in E^n$ is the state vector, $u \in E^m$ is the control vector, and the components of matrices A and B are constants that determine the properties of the object. Here we assume that the system (1.1) is completely controllable.

For the object of the form (1.1), we construct control in the form of linear feedback:

$$u = Kx \qquad (1.2)$$

We determine the integral quadratic functional in the solutions (1.1), (1.2) of the closed loop system:

$$J = J(K) = \int_0^\infty [x'Qx + u'Ru]\,dt, \qquad (1.3)$$

characterizing the behavior of the closed system

$$\dot{x} = (A + \mathrm{BK})x$$

Here, Q≥0 is a non-negative definite symmetric matrix and R>0 is a positive definite symmetric matrix. Matrices Q and R are weight matrix factors.

Then the problem of finding the LQR-optimal control can be formulated as follows:

$$J = J(\mathbf{K}) \to \min_{\mathbf{K} \in \Omega_K} \qquad (1.4)$$

where $\mathbf{\Omega}_K -$ is the set of constant K matrices such that the roots of the characteristic polynomial $\det(\boldsymbol{E}s - \boldsymbol{A} + \boldsymbol{B}\boldsymbol{Q}^{-1}\boldsymbol{B}^T\boldsymbol{S})$ /l of the closed system lie in the open left complex half-plane, i.e.

$$\boldsymbol{Re}\,\lambda_i(\boldsymbol{A} + \boldsymbol{BK}) < 0$$

To find the matrix $\mathbf{K}_0 = \arg\min_{\mathbf{K} \in \Omega_K}$ (1.2) in the optimal control, you need to perform the following steps:

1. Create the matrix algebra Riccati equation

$$-\frac{1}{d}SBQ^{-1}B^TS + A^TS + SA + C^TRC = 0$$

its solution is an S-symmetric positive definite matrix: the polynomial $\det(Es - A + BQ^{-1}B^TS)$ is Hurwitz, all its roots are in the left half-plane.

2. Based on the state vector $u = K_0x$, build a matrix of coefficients $K_0 = -Q^{-1}B^TS$ of the optimal controller.

8Performing these actions leads to the stabilization of the control, which is optimal in terms of functional (1.3) for the given Q and R matrices. As a rule, such matrices are not specified in the initial formulation of the problem, accordingly, the LQR optimization method involves the selection of elements of these matrices as an additional step in solving the problem. The specified matrices should be chosen in such a way that they reflect the requirements for dynamic processes in a closed system. In the general case, such a procedure is not formalized, and this allows us to pose the problem of choosing the matrices Q and R in such a way that the closed system satisfies the given requirements.

Let us now consider another optimization approach; its description can be found in the monograph [1], in which the requirements for the quality of dynamic processes are set by taking into account the restrictions directly imposed on the controlled variables by specifying the area to which they should belong. If the controlled variables belong to the specified areas, this means that the requirements for process dynamics are met. To define such a region, it is necessary to specify two time functions so that

$$x_2(t) < x_1(t),\ \forall t \in [0, T].$$

Let the right side of the closed-loop system model include a number of parameters combined into a common vector h, on which its behavior depends. $x(t, \boldsymbol{h})$ be some dynamic controlled variable (of interest to us), then a dynamic process can be considered admissible if the vector of adjustable parameters $\boldsymbol{h} \in E^p$ is chosen so that the inequalities are satisfied:

$$x_2(t) \le x(t, \boldsymbol{h}) \le x_1(t)\ \ \forall t \in [0, T] \qquad (1.5)$$

This is illustrated graphically in the following figure 2.

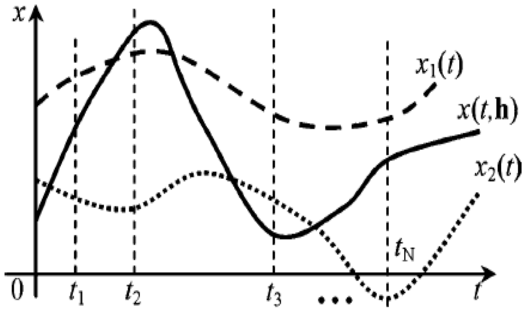

Fig.2. Allowable region for the characteristic x(t,h) of a dynamic process [1].

It can be seen that the admissible region defined by the pair $[\,x_1(t), x_2(t)\,]$, can limit the dynamic process x(t,h). To obtain an admissible dynamic process, it is necessary to select the vector h so that the simulated process in terms of the variable x(t,h) lies inside the domain, that is, to ensure that restrictions (1.5) are met.

An effective way to achieve this result is to formulate and solve an optimization problem; let's consider it. Let us fix a certain set of time moments from the interval on which we perform the simulation $t_i \in [0, T]$, $i = \overline{1, N}$. For each point, we will define the function $\alpha_i(\boldsymbol{h})$,, which will serve as a measure of the exit of the variable x(t,h) beyond the permissible region at time t_i (by penalty), using the formula

$$\alpha_i(h) = \begin{cases} 0,\ if\ x_2(t_i) \le x(t_i,\ h) \le x_1(t_i); \\ x(t_i,\ h) - x_1(t_i),\ if\ x(t_i,\ h) > x_1(t_i); \\ x_2(t_i) - x(t_i,\ h),\ if\ x(t_i,\ h) < x_2(t_i). \end{cases} \qquad (1.6)$$

Then the total penalty for leaving the corridor on the entire segment [0,T] will be determined by the sum:

$$\alpha(h) = \sum_{i=1}^{N} \alpha_i(h).$$

Based on the desire to minimize the total penalty, we can move on to the optimization problem

$$\widetilde{J}\,(h) = \alpha(h) \to min_{h \in E^P} \qquad (1.7)$$

This problem is solved numerically, and this means that an important feature and advantage of this approach is that when modeling and obtaining the value of the minimized function α(h), a nonlinear model of a closed-loop system can be used, containing any elements necessary for modeling [3].

For calculations, the work combines the capabilities of both described optimization approaches (in this and the previous paragraphs).

For this purpose, the requirements for dynamic processes in a closed system are characterized by specifying an admissible region on the plane into which the controlled dynamic variable must fall, and the vector of parameters h is composed of the parameters in the matrices Q and R in the functional (1.3). For simplicity, only diagonal elements will be considered non-zero elements of the specified matrices. In this case, these elements will set weighting coefficients in front of the corresponding state and control variables in the functional (1.3), determining the contribution of each such variable to the functional. We will consider the diagonal values of the matrix R to be positive fixed real numbers (the matrix R must be positive definite). The varying diagonal elements of the matrix Q (part of it may be fixed initially) are specified as the squares of some real numbers (parameters) that are not initially specified. All such parameters are combined into a vector h in the functional (1.7), and are selected by solving the optimization problem (1.7).

Fuzzy logic is a multi-valued logic that allows you to determine intermediate values for such generally accepted estimates as yes or no, true or false, black or white, etc. Such expressions, which are determined using fuzzy logic, will be represented, for example, as follows: a little hot or quite cold. This method of description can be formalized mathematically.

One of the basic concepts of fuzzy logic is the concept of a fuzzy set. It is known that from classical mathematics, defining crisp sets means using well-defined values, and defining fuzzy sets means using indefinite ones. Let us assume that set A contains all numbers from 0 to 5, and subset B of set A ranges from 2 to 4. To characterize set B, a function $\mu_A(x)$ (membership function) can be specified that assigns a number from 0 to 1 to each element of set B and characterizes the degree of membership of a particular element of set A to fuzzy set B. Value 1 means that the element definitely belongs to fuzzy set B, respectively, 0 – definitely does not belong to fuzzy set B. Membership functions can be specified in different ways. For example, a popular option is triangular membership functions, which are given as [4]:

$$trimf(x,a,b,c) = \begin{cases} 0, x \le a \\ \dfrac{x-a}{b-a}, a \le x \le b \\ 0, c \le x \end{cases}$$

If we set a=20, b=40, c=60, then we get the following graph:

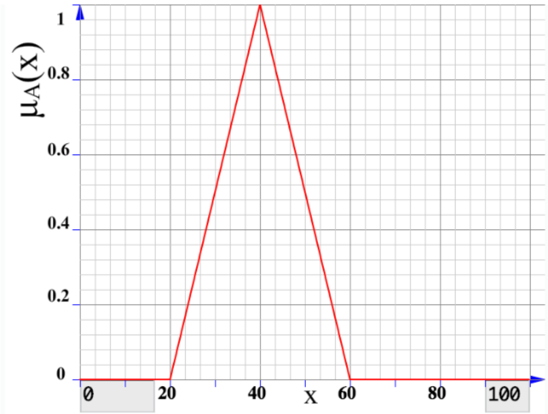

Fig.3. Membership function example.

Another example: if it's hot in the room, then we'll turn on the air conditioning. But how do we determine that the room is hot and when to turn on the air conditioning? Then, using fuzzy logic, the values are obtained: if it is a little hot, then we open the air conditioner slightly, that is, the degree of membership in the interval [0,1] can be obtained, for example, 0.3. If it's hot, then we open the air conditioner more, which means that the degree can already be 0.5. And if it's hot enough, then we open the air conditioner loudly. This already means that the membership level is maximum, that is, the value of the membership function is 1. The general procedure for using fuzzy sets involves the following stages: first, fuzzification or transition to fuzzification should be performed, setting rules and membership functions for specific variables, and, finally, defuzzification or elimination of fuzziness to obtain a specific specific value of a variable.

It is known that in fuzzy logic it is necessary to set several rules of this type, such as "IF...THEN...". This corresponds, for example, to the Mamdani method, that is, after THEN... we pass the value 1 or 0 (true or false). Let us set as a condition simply the equality of the states to some desired values.

And the idea of the Takagi-Sugeno method assumes that in the "IF...THEN..." rule, some mathematical expression is explicitly specified in the "...THEN..." condition. This means that there is no defuzzification step in this case.

This method can be used to construct a fuzzy TS model, which replaces the original nonlinear model.

These sets of "IF...THEN..." represent a replacement of a nonlinear system so that each rule corresponds to a replacement of a nonlinear system by one local linear subsystem, and the entire fuzzy system replacing the original nonlinear system is represented as a combination of all local subsystems:

R^i: IF $z_1(t)$ is M_1^i and … and $z_p(t)$ is M_p^i ,
THEN
$\dot{x}(t) = \boldsymbol{A}_i x(t) + B_i u(t), i = 1,2, \dots, r$

where M_j^i are fuzzy sets, (A_i, B_i) are a pair of matrices corresponding to the set (X_{ei}, U_{ei}), z_j(t) are dynamic variables that specify a specific equilibrium point in the rules.

Now let's define the membership functions, which in fuzzy logic represent the degree of membership of the variable under consideration to a given fuzzy set. Let $M_j^i(z(\mathrm{t}))$ denote that the variable $z_j(\mathrm{t})$ corresponds to the membership function the th rule.

Taking the product, we get:

$$W_i\big(z(t)\big) = \prod_{j=1}^{P} M_j^i(z(\mathrm{t}))$$

$W_i\big(z(t)\big)$ represents the weight of the i-th rule.

The weighted average method is widely used in industrial control; we use it and obtain the equations of the general system (TS-model), which we will consider instead of the original nonlinear system (2) when synthesizing control:

$$\dot{x} = \frac{\sum_{i=1}^{n} W_i\big(\boldsymbol{A}_i x(t) + \boldsymbol{B}_i u(t)\big)}{\sum_{i=1}^{n} W_i} \qquad (1.8)$$

Since the TS-model (1.8) is built on the basis of several linear models, its use instead of the original nonlinear model when constructing a controller means that you can try to reduce this problem to the use of a standard mathematical apparatus for working with individual linear systems that define the model (1.8) .

3.CONCLUSIONS

This paper considers the problem of controlling the movement of a car along a desired trajectory; the idea of synthesizing a fuzzy controller based on a fuzzy TS model in accordance with (1), (2) is used to solve it. This idea can be quite successfully implemented, and the LQR approach can be used to synthesize local controls, which will allow changing parameters and influencing control results. Accordingly, a simple version of the regulator was built and shortcomings were found. Then the rules were developed from the simple version, so that we managed to improve the result on the trajectory. Also, in the experiment, it was also possible to improve the results for the rule base, without formally checking the global asymptotic stability. Thus, this approach can be successfully applied in certain situations, however, improving the behavior of an object by increasing the number of local subsystems used entails a significant increase in the amount of computational work. There is also no guarantee of the applicability of the regulator in question to any acceptable conditions.

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